Homework 4: Algebraic identities, rationalization and Pythagoras theorem.

HW is Due Oct 12th

Basic algebraic identities for refreshing your memory:

Exponents Laws

If a *and* b are real numbers and n is a positive integer

$(ab)^n = a^n b^n$ (eq. 1)

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ (eq. 2)

$(a+b)^2 = a^2 + 2ab + b^2$ (eq.3)

$(a-b)^2 = a^2 - 2ab + b^2$ (eq.4)

And also: $a^2 - b^2 = (a-b)(a+b)$ (eq. 5)

Replacing in the last equality ***a*** by $\sqrt{\boldsymbol{a}}$, **b** by $\sqrt{\boldsymbol{b}}$, we get : $a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$ (eq. 6)

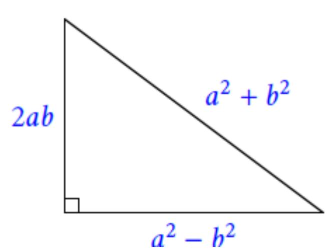
Also:

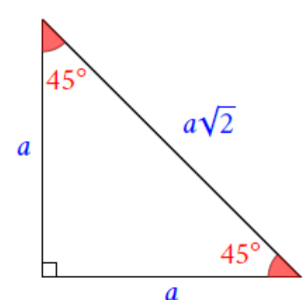
$$a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)$$

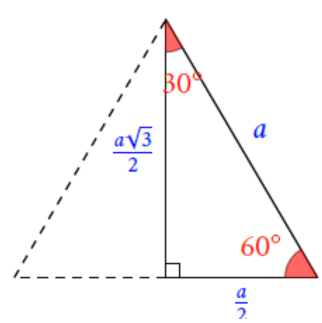
1. **Simplifying expressions with roots** (rational expressions)
   The above identity (eq. 6) can be used to simplify expressions with roots by expanding the fractions with a term which "removes" the roots from the denominator:
   $$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{\left(\sqrt{2}\right)^2 - 1^2} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$$

2. **Quadratic equations of a specific form**
   We also discussed solving simple equations:
   - linear equation (i.e., equation of the form $ax + b = 0$, with a, b some numbers, and x the unknown and equation)
   - two types of quadratic equations (i.e, equations where the unknown is squared, x^2) when the left-hand side could be factored as product of linear factors, i.e, $(x-2)(x+3) = 0$.

3. **Pythagoras' theorem:**
   In a right triangle with legs ***a*** and ***b***, and hypotenuse ***c***, the square of the hypotenuse is the sum of squares of each leg: $c^2 = a^2 + b^2$. The converse is also true, if the three sides of a triangle satisfy $c^2 = a^2 + b^2$, then the triangle is a right triangle. Some Pythagorean triples are: (3,4,5), (5,12,13), (7,24,25), (8.15,17), (9,40,41), (11,60,61), (20,21,29).
   To generate such Pythagorean triples, choose two positive integers a and b. Then plug the values into the sides as shown on the first picture:







Try to figure out again why the sides of this triangle satisfy the Pythagoras' Theorem!

45-45-90 Triangle: If one of the angles in a right triangle is 45^0, the other angle is also 45^0, and two of its legs are equal. If the length of a leg is a, the hypothenuse is $a\sqrt{2}$.

30-60-90 Triangle: If one of the angles in a right triangle is 30^0, the other angle is 60^0. Such triangle is a half of the equilateral triangle. That means that if the hypothenuse is equal to a, its smaller leg is equal to the half of the hypothenuse, i.e. $\frac{a}{2}$. Then we can find the other leg from the Pythagoras' Theorem, and it will be equal to $\frac{a\sqrt{3}}{2}$.

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Solve for x (i.e. find values of x that satisfies the following equations):
   a. $x^2 – 9x = -18$
   b. $x^2 - x = 3/4$
   c. $4x^2 – 49p^2q^2 = 0$
   d. $x^2 + 5x = 11/4$
   e. $x^2 + 3x = -5/4$

2. Write each of the following expressions with rational numbers in the denominator (i.e no roots in the denominator):

   a. $\frac{3-\sqrt{5}+\sqrt{7}}{3-\sqrt{5}-\sqrt{7}}$

   d. $\frac{\sqrt{p+q}-\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}}$

   b. $\frac{2+4\sqrt{7}}{2\sqrt{7}-1}$

   c. $\frac{a-\sqrt{a-1}}{a+\sqrt{a-1}}$

3. In a trapezoid ABCD with bases AD and BC, $\angle A = 90^0$, and $\angle D = 45^0$. It is also known that $AB = 10$ cm, and $AD = 3BC$. Find the area of the trapezoid.

4. In a right triangle ABC, BC is the hypotenuse. Draw AD perpendicular to BC, where D is on BC. The length of BC=13, and AB=5.What is the length of AD?