

Homework 3: Algebraic identities, simplificationHW is Due Oct 5th

Basic algebraic identities for refreshing your memory:

Exponents LawsIf a and b are real numbers and n is a positive integer

$$(ab)^n = a^n b^n \quad (\text{eq. 1})$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (\text{eq. 2})$$

$$(a + b)^2 = a^2 + 2ab + b^2 \quad (\text{eq.3})$$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad (\text{eq.4})$$

$$\text{And also: } a^2 - b^2 = (a - b)(a + b) \quad (\text{eq. 5})$$

$$\text{Replacing in the last equality } a \text{ by } \sqrt{a}, b \text{ by } \sqrt{b}, \text{ we get: } a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) \quad (\text{eq. 6})$$

Also:

$$\begin{aligned} a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \end{aligned}$$

Simplifying expressions with roots (rational expressions): I did not get time to discuss this in class. This example should help you do HW problems:

The above identity (eq. 6) can be used to simplify expressions with roots by expanding the fractions with a term which “removes” the roots from the denominator:

$$\frac{1}{\sqrt{2} + 1} = \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{(\sqrt{2})^2 - 1^2} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

Homework problems

Instructions: Please always write solutions on a **separate sheet of paper**. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Algebraic Lowest Common Multiple (LCM)

a. $3x, 9x^2$

b. $(x + y), (x^2 - y^2)$

c. a^2b, ab^2

d. $(x + 3), (x^2 + 6x + 9)$

e. $x^2 - 4, x^2 - 2x$

2. Simplify (take LCM of denominator and proceed)

a) $\frac{1}{a} + \frac{1}{b}$

d) $\frac{a}{b} + \frac{b}{c} + \frac{c}{d}$

b) $\frac{3}{x} + \frac{5}{xy} + \frac{5}{3a}$

c) $\frac{x}{(x^2 - y^2)} - \frac{y}{(x + y)^2}$

e)
$$\begin{aligned} &\frac{a+b}{(b-c)(c-a)} + \\ &\frac{b+c}{(c-a)(a-b)} + \\ &\frac{c+a}{(a-b)(b-c)} \end{aligned}$$

3. Write each of the following expressions in the form $a + b\sqrt{3}$ with rational a, b. (No root in the denominator)

a. $\frac{1}{1-2\sqrt{3}}$

b. $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

c. $\frac{1+2\sqrt{3}}{\sqrt{3}}$