

Homework 2: Algebraic identities, simplification and factorization.HW is Due Oct 6<sup>th</sup>

Basic algebraic identities for refreshing your memory:

**Exponents Laws**If  $a$  and  $b$  are real numbers and  $n$  is a positive integer

$$(ab)^n = a^n b^n \quad (\text{eq. 1})$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (\text{eq. 2})$$

$$(a + b)^2 = a^2 + 2ab + b^2 \quad (\text{eq.3})$$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad (\text{eq.4})$$

$$\text{And also: } a^2 - b^2 = (a - b)(a + b) \quad (\text{eq. 5})$$

$$\text{Replacing in the last equality } \mathbf{a} \text{ by } \sqrt{a}, \mathbf{b} \text{ by } \sqrt{b}, \text{ we get : } a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) \quad (\text{eq. 6})$$

Also:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Homework problems

**Instructions:** Please always write solutions on a **separate sheet of paper**. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Using algebraic identities calculate

a.  $299^2 + 598 + 1$       d.  $16^3 - 15^3$

b.  $199^2$       e.  $57^3 - 56^3$

c.  $51^2 - 102 + 1$

2. Expand

a.  $(4a - b + c)^2 =$

b.  $(a + 9)(a - 9) =$

c.  $(3a - 2b)^4 =$

3. Factorize (i.e., write as a product) the following expressions:

a.  $(a - b)^2 - (b + c)^2$

b.  $a^4 + 4y^4$

c.  $x^4 + 4 \cdot 10^{8n}$

d.  $x^4 - 8x^2y^2 + 4y^4$

e.  $3p^2 - 20p + 32$

f.  $22x^2 + 47x + 21$

g.  $8m^3 - 27p^3q^3$

h.  $21b^2 - 37b - 28$

i.  $27x^3 - y^3$

j.  $24z^2 - 103z + 55$

k.  $9z^4 - 148z^2 + 64$