MATH 7:
TRIGONOMETRIC EQUATIONS

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We can use our experience solving linear and quadratic equations to solve analogous problems involving trigonometric functions.

There are important differences, however. For example, the periodicity of the trigonometric functions usually means that there are many solutions to an equation (an infinite number!). Let's look at a few examples:

Solving the equation $\sin x = \sin c$**.** Using the trigonometric circle (see figure 1), we see that, for a given c, the solution to the equation $\sin x = \sin c$ is

$$x = c + 360^\circ \times n$$
or
$$x = \pi - c + 360^\circ \times n,$$

where n can be any integer number (that's just because adding full turns doesn't change the sign!), which is why we have an infinite number of solutions. We can use the same technique to solve equations of the type $\sin x = a$. For example, equation $\sin x = \frac{\sqrt{3}}{2}$ has solutions

$$x = 60^\circ + 360^\circ \times n$$
or
$$x = 120^\circ + 360^\circ \times n,$$

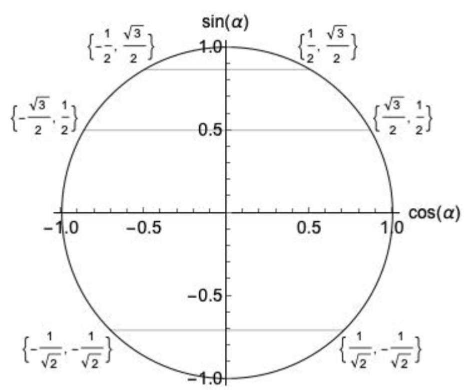

FIGURE 1. A few examples of the relation $\sin x = \sin(180^\circ - x)$.

Solving the equation $\cos x = \cos c$**.** By a similar inspection of the trigonometric circle (figure 2), we see that, for a given c, the solution to the equation $\cos x = \cos c$ is

$$x = c + 360^\circ \times n$$
or
$$x = -c + 360^\circ \times n,$$

where n can be any integer number. We can use the same technique to solve equations of the type $\cos x = a$. For example, equation $\cos x = \frac{\sqrt{2}}{2}$ has solutions

$$x = 45^\circ + 360^\circ \times n$$

or

$$x = -45^\circ + 360^\circ \times n,$$

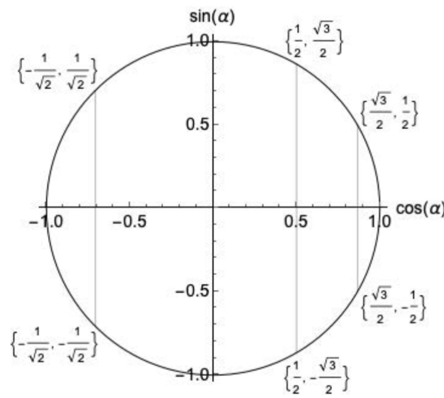

FIGURE 2. A few examples of the relation $\cos x = \cos(-x)$.

General Strategy. We see from these examples that, rather than trying to memorize all different cases, *one should always refer to the trigonometric circle*, from which the answer can be read off immediately. This is also true for trigonometric inequalities.

CLASSWORK

1. Solve the following equations
   (a) $\sin x = \sin \frac{\pi}{5}$
   (b) $\sin x = 0$
   (c) $\cos x = \frac{\sqrt{2}}{2}$
   (d) $\cos x = \frac{1}{2}$
   (e) $\cos(x + \frac{\pi}{6}) = 0$