MATH 7: HANDOUT 20

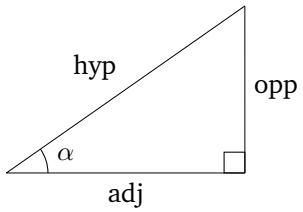
TRIGONOMETRY I: BASIC DEFINITIONS (SUMMARY)

Defining Sine, Cosine, and Tangent

In a right triangle with acute angle α:

Definition 1 (Trigonometric Ratios)**.**

$$\sin\alpha = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos\alpha = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \tan\alpha = \frac{\text{Opposite}}{\text{Adjacent}}$$



Key insight: The ratios depend *only on the angle*, not on the size of the triangle. Why? All right triangles with the same acute angle are similar, so side ratios are constant.

Also: $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$.

Scaling: Any Right Triangle

If the hypotenuse has length r, then by the definitions above:

$$\text{opposite} = r\sin\alpha, \qquad \text{adjacent} = r\cos\alpha.$$

This is extremely useful: if you know the hypotenuse and angle, you immediately get both legs!

Solving Right Triangles

If you know one side and one acute angle, you can find any other side. Choose the right function:

- **Opposite and Hypotenuse** $\to$ use $\sin$
- **Adjacent and Hypotenuse** $\to$ use $\cos$
- **Opposite and Adjacent** $\to$ use $\tan$

Example: Ladder (4 m) at 65 angle. Height?

$$\sin 65 = \frac{h}{4} \;\Rightarrow\; h = 4\sin 65 \approx 3.63 \text{ m}$$

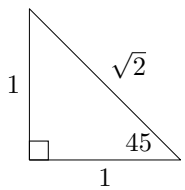
Example: Tower 20 m away, elevation 50. Height?

$$\tan 50 = \frac{h}{20} \;\Rightarrow\; h = 20\tan 50 \approx 23.8 \text{ m}$$

Special Angles

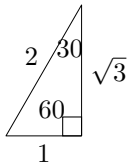
These exact values come from two simple geometric constructions:

The 45 **angle:** In an isosceles right triangle with legs $= 1$, the hypotenuse $= \sqrt{2}$.



$\sin 45 = \cos 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

The $30/60$ **angles:** Split an equilateral triangle (side 2) with an altitude $\to$ sides 1, $\sqrt{3}$, 2.



$\sin 30 = \frac{1}{2}$, $\sin 60 = \frac{\sqrt{3}}{2}$

Angle	0	30	45	60	90
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

The extreme cases:

- As $\alpha \to 0$: the opposite side shrinks to 0, adjacent $\to$ hypotenuse. So $\sin 0 = 0$, $\cos 0 = 1$.
- As $\alpha \to 90$: the adjacent side shrinks to 0, opposite $\to$ hypotenuse. So $\sin 90 = 1$, $\cos 90 = 0$.
- $\tan 90$ is undefined because it would require dividing by 0 (adjacent side $= 0$).

Memory tips:

- $\sin$ values for $0, 30, 45, 60, 90$: $\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}$
- $\cos$ values are the same sequence in reverse order

Common Mistakes

- **Confusing opposite/adjacent:** Always identify which angle you're using first.
- **Calculator mode:** Make sure it's in degree mode, not radians!

Homework

Problems marked with M *or unmarked are expected from every student. Problems marked with* H *are optional challenge problems.*

1. Which quantity is greater?
   (a) 0 or $\sin 0$
   (b) 1 or $\sin 30$
   (c) $\sin 45$ or $\cos 45$
   (d) $\cos 60$ or $\sin 30$
2. From the top of a 50 m lighthouse, the angle of depression to a boat is 12. How far is the boat from the shore?
3. A building of unknown height casts a 20 m shadow when the sun's angle of elevation is 53. Find the height of the building.
4. A ladder of length L rests on a ledge whose height is one half of the ladder's length. The ladder makes a 45 angle with the ground.
   (a) How long is the segment of the ladder between the ground and the point where it meets the ledge?
   (b) How much higher is the top of the ladder above the ledge?
5. A cruise ship sails 3 miles due north, then turns northwest and travels another 3 miles. How far is it from its starting point? *(Northwest is 45 west of north.)*
6. An airplane flies 100 km due north, then turns 60 to the east and flies another 80 km. How far east and how far north is it from the starting point?
7. M Let $ABCD$ be a parallelogram with $AB = 1$, $AD = 3$, and $\angle A = 40$. Find the lengths of both diagonals of the parallelogram.
8. H Prove that the area of a triangle $\triangle ABC$ can be computed using the formula
$$A = \frac{1}{2}\, AB \cdot AC \cdot \sin \angle A.$$
(Hint: express the altitude from B in terms of AC and $\sin \angle A$.)
9. A regular 12-gon is inscribed in a circle of radius 1. Find the side length. *(Hint: the central angle is 30.)*
10. M What is the area of a regular pentagon inscribed in a circle of radius 10? *(Your answer should use trigonometric functions.)*