

MATH 7: HANDOUT 19

COORDINATE GEOMETRY III: PARABOLAS (SUMMARY)

Quadratic Functions and Parabolas

A **parabola** is the graph of a quadratic function $y = ax^2 + bx + c$ (where $a \neq 0$).

Key forms:

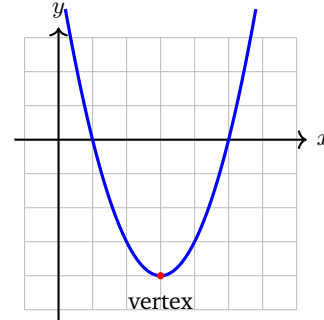
- **Standard form:** $y = ax^2 + bx + c$
- **Vertex form:** $y = a(x - h)^2 + k$

Convert standard to vertex form by **completing the square**.

Example: $y = x^2 - 6x + 5$

$$= (x^2 - 6x + 9) - 9 + 5 = (x - 3)^2 - 4$$

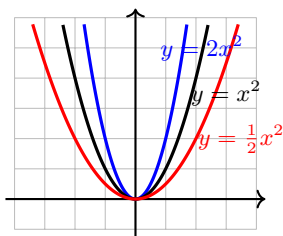
Vertex: $(3, -4)$



Properties of Parabolas

Property	Formula/Rule
Vertex (from vertex form)	(h, k) for $y = a(x - h)^2 + k$
Vertex x -coordinate (from standard)	$x = -\frac{b}{2a}$
Axis of symmetry	$x = h$ (vertical line through vertex)
Opens upward	$a > 0$ (vertex is minimum)
Opens downward	$a < 0$ (vertex is maximum)
y -intercept	c (set $x = 0$)
x -intercepts	Solve $ax^2 + bx + c = 0$

How $|a|$ Affects Width



- $|a| > 1$: **narrower** (steeper)
- $|a| < 1$: **wider** (flatter)
- $|a| = 1$: same as $y = x^2$

Finding x -Intercepts Using Symmetry

Since parabolas are symmetric about their axis:

- If you know one x -intercept and the axis of symmetry, reflect to find the other.
- **Example:** Axis at $x = 4$, one intercept at $x = 1 \Rightarrow$ other intercept at $x = 7$.

Discriminant and Number of x -Intercepts

For $ax^2 + bx + c = 0$, the discriminant $D = b^2 - 4ac$ determines:

- $D > 0$: Two distinct x -intercepts
- $D = 0$: One x -intercept (vertex touches x -axis)
- $D < 0$: No x -intercepts

Focus and Directrix (Geometric Definition)

A parabola is the set of points equidistant from a point (the **focus**) and a line (the **directrix**).

For $y = ax^2$: Focus: $\left(0, \frac{1}{4a}\right)$ Directrix: $y = -\frac{1}{4a}$

This property explains why parabolic mirrors focus light to a single point.

Key Formulas Summary

Concept	Formula
Standard form	$y = ax^2 + bx + c$
Vertex form	$y = a(x - h)^2 + k$, vertex at (h, k)
Completing the square	$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$
Vertex x -coordinate	$x = -\frac{b}{2a}$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Vieta's formulas	$x_1 + x_2 = -\frac{b}{a}, \quad x_1x_2 = \frac{c}{a}$

Common Mistakes to Avoid

- **Sign error in vertex:** If $y = (x + 2)^2 - 3$, vertex is $(-2, -3)$, not $(2, -3)$.
- **Forgetting a when completing square:** For $y = 2x^2 + 8x$, factor first: $2(x^2 + 4x) = 2(x + 2)^2 - 8$.
- **Width confusion:** Larger $|a|$ means *narrower*, not wider.
- **Vertex vs. roots:** Vertex gives max/min point, not x -intercepts.

Homework

- Write each quadratic in vertex form $y = a(x - h)^2 + k$ and identify the vertex:
 - $y = x^2 + 8x + 12$
 - $y = 2x^2 - 12x + 10$
- Sketch the graph of $y = -(x - 3)^2 + 4$. Label the vertex and find the x -intercepts.
- Find the minimum or maximum value of each function:
 - $f(x) = x^2 - 10x + 21$
 - $g(x) = -x^2 + 4x + 5$
- Write the equation of a parabola with vertex $(3, -1)$ that passes through $(5, 7)$.
- M** The profit P (in dollars) from selling x items is given by $P(x) = -2x^2 + 80x - 300$. Find:
 - How many items should be sold to maximize profit?
 - What is the maximum profit?
- M** A projectile is launched from ground level with height $h(t) = -16t^2 + 64t$ feet after t seconds.
 - When does the projectile reach its maximum height?
 - What is the maximum height?
 - When does the projectile return to the ground?
- M** For what values of k does the equation $x^2 - 6x + k = 0$ have:
 - Two distinct real roots?
 - Exactly one real root?
 - No real roots?
- M** Find all points where the line $y = 2x - 3$ intersects the parabola $y = x^2 - 2x + 1$.
- H** Find all values of m for which the line $y = mx - 1$ is tangent to the parabola $y = x^2$.
Hint: A line is tangent to a parabola when they intersect at exactly one point.
- H** Prove that for any point P on the parabola $y = \frac{x^2}{4} + 1$, the distance from P to the x -axis equals the distance from P to $(0, 2)$.