

MATH 7: HANDOUT 13

QUADRATIC EQUATIONS II

Vieta's Formulas

Sometimes we need to find an expression involving the roots of a quadratic equation, without explicitly calculating those roots. In such cases, the **Vieta formulas** are extremely useful: they relate the coefficients of a quadratic equation directly to the sum and product of its roots.

Who was François Viète (1540–1603)?

François Viète (also spelled Vieta) was a French mathematician, lawyer, and royal advisor — one of the brightest minds of the late Renaissance. He served as a counsellor to King Henry IV of France and famously **broke secret ciphers** used by Spain and the papal court, earning him renown as one of the first professional cryptanalysts in Europe.

In mathematics, Viète transformed algebra from a collection of rules into a symbolic language. Before him, equations were written mostly in words, and each problem was treated as unique. Viète introduced the systematic use of **letters for both known and unknown quantities**, laying the groundwork for modern algebraic notation. His 1591 book *In artem analyticam isagoge* (*Introduction to the Analytic Art*) marks the beginning of symbolic algebra as we know it today. He was also among the first to observe that the coefficients of a polynomial encode precise relationships among its roots — the discovery now known as the **Vieta formulas**. These relationships make it possible to work with roots without solving equations directly.



François Viète

Beyond algebra, Viète made a remarkable contribution to analysis by finding the first known **infinite product for π** :

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2} \dots$$

or equivalently,

$$\pi = 2 \times \frac{2}{\sqrt{2}} \times \frac{2}{\sqrt{2+\sqrt{2}}} \times \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \times \dots$$

This elegant expression, derived in 1593, was centuries ahead of its time — connecting geometry, trigonometry, and the emerging concept of infinite limits.

The Formulas

Consider the general quadratic equation:

$$ax^2 + bx + c = 0$$

with roots x_1 and x_2 . Then:

$$\begin{aligned}x_1 + x_2 &= -\frac{b}{a} \\x_1 x_2 &= \frac{c}{a}\end{aligned}$$

In the special case when $a = 1$, these simplify to:

$$\begin{aligned}x_1 + x_2 &= -b \\x_1x_2 &= c\end{aligned}$$

Why They Work

If an equation $p(x) = 0$ has a root x_1 , then $p(x)$ is divisible by $(x - x_1)$, i.e.

$$p(x) = (x - x_1)q(x)$$

for some polynomial $q(x)$. If both roots x_1 and x_2 are known, then

$$ax^2 + bx + c = a(x - x_1)(x - x_2).$$

Expanding this expression gives:

$$ax^2 + bx + c = a(x^2 - (x_1 + x_2)x + x_1x_2),$$

and by comparing coefficients, we obtain the two formulas above.

Example 1: Checking the Formula. Let $2x^2 - 6x + 2.5 = 0$. Its roots are $x_1 = 0.5$ and $x_2 = 2.5$. Then:

$$2x^2 - 6x + 2.5 = 2(x - 0.5)(x - 2.5).$$

Indeed,

$$x_1 + x_2 = 3 = -\frac{b}{a}, \quad x_1x_2 = 1.25 = \frac{c}{a}.$$

□

Example 2: Using Vieta Formulas Without Solving. Find $x_1^2 + x_2^2$ if x_1 and x_2 are roots of $x^2 - 5x + 6 = 0$.

We know from Vieta:

$$x_1 + x_2 = 5, \quad x_1x_2 = 6.$$

Then:

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 5^2 - 2 \cdot 6 = 13.$$

No need to find the actual roots!

Example 3: Inverse and Fractional Expressions. If x_1 and x_2 are roots of $2x^2 - 3x + 4 = 0$, find $\frac{1}{x_1} + \frac{1}{x_2}$.

We use:

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1x_2}.$$

By Vieta:

$$x_1 + x_2 = \frac{3}{2}, \quad x_1x_2 = 2.$$

Hence:

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{(3/2)}{2} = \frac{3}{4}.$$

Example 4: Creating a New Equation. If x_1 and x_2 are roots of $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are $x_1 + 1$ and $x_2 + 1$.

We compute:

$$(x_1 + 1) + (x_2 + 1) = (x_1 + x_2) + 2 = 9,$$

$$(x_1 + 1)(x_2 + 1) = x_1x_2 + (x_1 + x_2) + 1 = 10 + 7 + 1 = 18.$$

Thus the new equation is:

$$x^2 - 9x + 18 = 0.$$

Quick Check

1. For the equation $x^2 - 8x + 15 = 0$, find the sum and product of the roots without solving.
2. If x_1 and x_2 are roots of $3x^2 + 6x - 9 = 0$, find $x_1 + x_2$ and x_1x_2 .
3. Find $\frac{1}{x_1} + \frac{1}{x_2}$ if x_1, x_2 are roots of $x^2 - 4x + 3 = 0$.

Symmetric Expressions

An expression in x_1 and x_2 is called **symmetric** if it does not change when we swap the two variables. For example, $x_1 + x_2$ is symmetric because swapping gives $x_2 + x_1$, which is the same thing. Similarly, x_1x_2 is symmetric because $x_2x_1 = x_1x_2$.

Here are more examples of symmetric expressions:

$$x_1^2 + x_2^2, \quad x_1^3 + x_2^3, \quad \frac{1}{x_1} + \frac{1}{x_2}, \quad (x_1 - x_2)^2.$$

In contrast, $x_1 - x_2$ is *not* symmetric: swapping gives $x_2 - x_1 = -(x_1 - x_2)$, which is different. However, $(x_1 - x_2)^2$ is symmetric, because $(x_2 - x_1)^2 = (x_1 - x_2)^2$.

Why do symmetric expressions matter? When we solve a quadratic equation, we get two roots — but there is no natural way to decide which one is “ x_1 ” and which is “ x_2 .” The quadratic formula gives us

$$x = \frac{-b + \sqrt{D}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{D}}{2a},$$

but either one could be called x_1 . This means that any *meaningful* quantity involving both roots — one that doesn’t depend on an arbitrary choice of labeling — must be symmetric.

For instance, if a problem asks: “Find the sum of the squares of the roots,” the answer must be symmetric, because the problem doesn’t specify which root is which.

The Fundamental Theorem of Symmetric Expressions. Here is a remarkable fact:

Every symmetric expression in x_1 and x_2 can be rewritten using only $x_1 + x_2$ and x_1x_2 .

This is why Vieta’s formulas are so powerful! Since we can compute $x_1 + x_2$ and x_1x_2 directly from the coefficients (without solving the equation), we can find *any* symmetric expression without ever finding the roots themselves.

Building a Toolkit. Let’s derive formulas for some common symmetric expressions. Write $s = x_1 + x_2$ and $p = x_1x_2$ for convenience.

Sum of squares:

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = s^2 - 2p.$$

Square of the difference:

$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = s^2 - 4p.$$

(Notice this equals the discriminant divided by a^2 — not a coincidence!)

Sum of cubes:

$$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = s^3 - 3ps.$$

Sum of reciprocals:

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1x_2} = \frac{s}{p}.$$

Example 5. Find $x_1^3 + x_2^3$ if x_1 and x_2 are roots of $x^2 - 4x + 1 = 0$.

From Vieta's formulas:

$$x_1 + x_2 = 4, \quad x_1x_2 = 1.$$

Using our formula for the sum of cubes:

$$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = 4^3 - 3 \cdot 1 \cdot 4 = 64 - 12 = 52.$$

Example 6. Find $\frac{1}{x_1^2} + \frac{1}{x_2^2}$ if x_1, x_2 are roots of $x^2 - 3x + 2 = 0$.

From Vieta's formulas:

$$x_1 + x_2 = 3, \quad x_1x_2 = 2.$$

We rewrite the expression by combining the fractions:

$$\frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{x_1^2 + x_2^2}{(x_1x_2)^2}.$$

We already know that $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 9 - 4 = 5$. Therefore:

$$\frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{5}{2^2} = \frac{5}{4}.$$

Summary of Key Ideas

- A **symmetric expression** is unchanged when x_1 and x_2 are swapped.
- For $ax^2 + bx + c = 0$ with roots x_1, x_2 :

$$x_1 + x_2 = -\frac{b}{a}, \quad x_1x_2 = \frac{c}{a}.$$

- **Any** symmetric expression in x_1 and x_2 can be computed using only $x_1 + x_2$ and x_1x_2 .
- This means we can evaluate symmetric expressions **without solving the equation**.

Quick Check

1. Which of the following are symmetric expressions?

(a) $x_1^2 + x_2^2$ (b) $x_1 - x_2$ (c) $x_1^2x_2 + x_1x_2^2$ (d) $\frac{x_1}{x_2}$

2. If $x_1 + x_2 = 6$ and $x_1x_2 = 8$, find $x_1^2 + x_2^2$.

3. If x_1, x_2 are roots of $x^2 - 5x + 2 = 0$, find $x_1^3 + x_2^3$.

Generalization to Higher Degrees (OPTIONAL)

Vieta's idea extends beautifully beyond quadratic equations. For a **cubic equation** (degree 3),

$$ax^3 + bx^2 + cx + d = 0,$$

with roots x_1 , x_2 , and x_3 , we have:

$$\begin{aligned}x_1 + x_2 + x_3 &= -\frac{b}{a}, \\x_1x_2 + x_1x_3 + x_2x_3 &= \frac{c}{a}, \\x_1x_2x_3 &= -\frac{d}{a}.\end{aligned}$$

Notice the pattern:

- The **sum of the roots** (taken one at a time) relates to the second coefficient.
- The **sum of all pairwise products** (taken two at a time) relates to the third coefficient.
- The **product of all roots** (taken three at a time) relates to the fourth coefficient.
- The signs alternate: $-$, $+$, $-$.

Example 7: Cubic Equation. Let

$$x^3 - 6x^2 + 11x - 6 = 0.$$

Then:

$$\begin{aligned}x_1 + x_2 + x_3 &= -\frac{-6}{1} = 6, \\x_1x_2 + x_1x_3 + x_2x_3 &= \frac{11}{1} = 11, \\x_1x_2x_3 &= -\frac{-6}{1} = 6.\end{aligned}$$

(Indeed, this equation has roots 1, 2, and 3 — check that $1 + 2 + 3 = 6$, that $1 \cdot 2 + 1 \cdot 3 + 2 \cdot 3 = 11$, and that $1 \cdot 2 \cdot 3 = 6$.)

For a **quartic equation** (degree 4),

$$ax^4 + bx^3 + cx^2 + dx + e = 0,$$

with roots x_1 , x_2 , x_3 , and x_4 , the pattern continues:

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= -\frac{b}{a}, \\(\text{sum of all products of two roots}) &= \frac{c}{a}, \\(\text{sum of all products of three roots}) &= -\frac{d}{a}, \\x_1x_2x_3x_4 &= \frac{e}{a}.\end{aligned}$$

The signs continue to alternate: $-$, $+$, $-$, $+$.

Example 8: Quartic Equation. If x_1, x_2, x_3, x_4 are roots of

$$x^4 - 4x^3 + 5x^2 - 2x + 7 = 0,$$

then:

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 4, \\x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 &= 5, \\x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 &= 2, \\x_1x_2x_3x_4 &= 7.\end{aligned}$$

Example 9: Using Symmetry for Cubics. Suppose x_1, x_2, x_3 are roots of $x^3 - 2x^2 + 3x - 4 = 0$. Find $x_1^2 + x_2^2 + x_3^2$.

Just as in the quadratic case, we can express this using Vieta's formulas. From the equation:

$$x_1 + x_2 + x_3 = 2, \quad x_1x_2 + x_1x_3 + x_2x_3 = 3.$$

Now we use a similar trick to the quadratic case. Expanding $(x_1 + x_2 + x_3)^2$ gives:

$$(x_1 + x_2 + x_3)^2 = x_1^2 + x_2^2 + x_3^2 + 2(x_1x_2 + x_1x_3 + x_2x_3).$$

Therefore:

$$x_1^2 + x_2^2 + x_3^2 = (x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_1x_3 + x_2x_3) = 2^2 - 2 \cdot 3 = 4 - 6 = -2.$$

Generalized Vieta's Formulas

For a polynomial of degree n with leading coefficient a_n :

- The sum of all roots equals $-(\text{second coefficient})/a_n$.
- The sum of all pairwise products of roots equals $+(\text{third coefficient})/a_n$.
- The sum of all triple products equals $-(\text{fourth coefficient})/a_n$.
- And so on, with alternating signs: $-, +, -, +, \dots$
- The product of all roots equals $(-1)^n \cdot (\text{constant term})/a_n$.

The same tricks for computing symmetric expressions (like $x_1^2 + x_2^2 + \dots$) work for any degree!

Key Takeaways

Common Mistakes to Avoid

- **Forgetting the negative sign:** The sum of roots is $-b/a$, not b/a . The minus sign is easy to forget!
- **Not rewriting in standard form:** Before applying Vieta's formulas, make sure the equation is written as $ax^2 + bx + c = 0$. For example, $3x^2 - x = 4$ must first be rewritten as $3x^2 - x - 4 = 0$.
- **Confusing sum and product:** Remember: $x_1 + x_2 = -b/a$ (sum uses b), and $x_1x_2 = c/a$ (product uses c).
- **Sign errors in higher degrees:** The signs alternate $-, +, -, +, \dots$ starting from the sum of roots. Double-check your signs!
- **Dividing by zero:** When computing $\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1x_2}$, make sure $x_1x_2 \neq 0$ (i.e., $c \neq 0$).

Quick Reference: Symmetric Expression Formulas

Let $s = x_1 + x_2$ and $p = x_1x_2$. Then:

Expression	Formula
$x_1^2 + x_2^2$	$s^2 - 2p$
$(x_1 - x_2)^2$	$s^2 - 4p$
$x_1^3 + x_2^3$	$s^3 - 3ps$
$\frac{1}{x_1} + \frac{1}{x_2}$	$\frac{s}{p}$
$\frac{1}{x_1^2} + \frac{1}{x_2^2}$	$\frac{s^2 - 2p}{p^2}$
$x_1^2x_2 + x_1x_2^2$	ps

Quick Check

- For the cubic $x^3 + 2x^2 - 5x + 3 = 0$ with roots x_1, x_2, x_3 , find:
 - $x_1 + x_2 + x_3$
 - $x_1x_2x_3$
- For a quartic equation, the coefficient pattern gives alternating signs $-, +, -, +$ for the four Vieta formulas. What sign appears in front of the product of all four roots?
- If x_1, x_2, x_3 are roots of $x^3 - 6x^2 + 11x - 6 = 0$, find $x_1^2 + x_2^2 + x_3^2$.

Classwork

1. Without solving, find the sum and product of the roots of each equation:

(a) $x^2 - 9x + 20 = 0$

(b) $2x^2 + 7x - 4 = 0$

2. Let x_1 and x_2 be the roots of $x^2 - 6x + 7 = 0$. Without solving, find:

(a) $x_1^2 + x_2^2$

(b) $\frac{1}{x_1} + \frac{1}{x_2}$

3. Without solving, determine whether the roots of $x^2 + 3x - 10 = 0$ are both positive, both negative, or of opposite signs. Explain your reasoning using Vieta's formulas.

4. Find a quadratic equation with integer coefficients whose roots are 4 and -2 .

5. Let x_1 and x_2 be the roots of $x^2 - 5x + 6 = 0$. Find the quadratic equation whose roots are $2x_1$ and $2x_2$.

6. Let x_1, x_2, x_3 be the roots of $x^3 - 4x^2 + 5x - 2 = 0$. Find:

(a) $x_1 + x_2 + x_3$

(b) $x_1x_2x_3$

Classwork Solutions

1. (a) For $x^2 - 9x + 20 = 0$: Here $a = 1$, $b = -9$, $c = 20$.

$$x_1 + x_2 = -\frac{b}{a} = -\frac{-9}{1} = 9, \quad x_1x_2 = \frac{c}{a} = \frac{20}{1} = 20.$$

- (b) For $2x^2 + 7x - 4 = 0$: Here $a = 2$, $b = 7$, $c = -4$.

$$x_1 + x_2 = -\frac{b}{a} = -\frac{7}{2}, \quad x_1x_2 = \frac{c}{a} = \frac{-4}{2} = -2.$$

2. For $x^2 - 6x + 7 = 0$, by Vieta's formulas: $x_1 + x_2 = 6$ and $x_1x_2 = 7$.

(a) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 6^2 - 2 \cdot 7 = 36 - 14 = 22$.

(b) $\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1x_2} = \frac{6}{7}$.

3. For $x^2 + 3x - 10 = 0$, by Vieta's formulas:

$$x_1 + x_2 = -3, \quad x_1x_2 = -10.$$

Since $x_1x_2 = -10 < 0$, the product of the roots is negative. This means one root is positive and one root is negative — the roots have **opposite signs**.

(Note: The sum being negative tells us the positive root has smaller absolute value than the negative root.)

4. If the roots are 4 and -2 , then by Vieta's formulas:

$$x_1 + x_2 = 4 + (-2) = 2, \quad x_1x_2 = 4 \cdot (-2) = -8.$$

The quadratic equation with these roots is:

$$x^2 - (x_1 + x_2)x + x_1x_2 = 0 \implies \boxed{x^2 - 2x - 8 = 0}.$$

5. For $x^2 - 5x + 6 = 0$, by Vieta's formulas: $x_1 + x_2 = 5$ and $x_1x_2 = 6$.

The new roots are $2x_1$ and $2x_2$. We find:

$$2x_1 + 2x_2 = 2(x_1 + x_2) = 2 \cdot 5 = 10,$$

$$(2x_1)(2x_2) = 4x_1x_2 = 4 \cdot 6 = 24.$$

The new equation is:

$$x^2 - 10x + 24 = 0.$$

6. For $x^3 - 4x^2 + 5x - 2 = 0$, using Vieta's formulas for cubics:

(a) $x_1 + x_2 + x_3 = -\frac{-4}{1} = 4$.

(b) $x_1x_2x_3 = -\frac{-2}{1} = 2$.

Homework

Difficulty: No mark = routine **M** = moderate **H** = challenging

1. Solve the following equations.

(a) $x^2 - 5x + 6 = 0$

(c) $\sqrt{2x+1} = x$

(b) $x^2 = 1 + x$

(d) $x + \frac{1}{x} = 3$

2. **M** Let x and y be two numbers with

$$x + y = a \quad \text{and} \quad xy = b.$$

Express each of the following using only a and b .

Example. We know that $(x + y)^2 = x^2 + 2xy + y^2$, so

$$x^2 + y^2 = (x + y)^2 - 2xy = a^2 - 2b.$$

(a) $(x - y)^2$

(d) $\frac{1}{x^2} + \frac{1}{y^2}$

(b) $x^3 + y^3$

(e) $x^4 + y^4$

(c) $\frac{1}{x} + \frac{1}{y}$

(f) $\frac{x}{y} + \frac{y}{x}$

3. Without solving, find the sum and product of the roots of each equation:

(a) $x^2 - 7x + 12 = 0$

(c) $3x^2 - x = 4$

(b) $2x^2 + 5x - 3 = 0$

(d) $x(x - 4) = 5$

4. Let x_1 and x_2 be the roots of the equation $x^2 + 5x - 7 = 0$. Find the following without solving for the roots explicitly.

(a) $x_1 + x_2$

(b) x_1x_2

(c) $x_1^2 + x_2^2$

(d) $(x_1 - x_2)^2$

5. **Signs of roots.** Without solving, determine whether the roots of each equation are both positive, both negative, or of opposite signs. Explain your reasoning.

(a) $x^2 - 5x + 6 = 0$

(c) $x^2 - 5x - 6 = 0$

(b) $x^2 + 5x + 6 = 0$

(d) $x^2 + 5x - 6 = 0$

Hint: What does the sign of $x_1 + x_2$ tell you? What does the sign of x_1x_2 tell you?

6. The numbers x and y satisfy

$$x + y = 5, \quad xy = 6.$$

(a) Find the value of $x^2 + y^2$.

(b) Find the value of $(x - y)^2$.

(c) Find all possible values of x and y .

Hint: What quadratic equation has x and y as its roots?

7. **M** If $x + \frac{1}{x} = 7$, find the values of

$$x^2 + \frac{1}{x^2} \quad \text{and} \quad x^3 + \frac{1}{x^3}.$$

8. **M** Find a quadratic equation with integer coefficients whose roots are:

(a) 3 and -5

- (b) $\frac{1}{2}$ and $\frac{2}{3}$
 (c) $2 + \sqrt{3}$ and $2 - \sqrt{3}$

9. **M** Let x_1 and x_2 be the roots of $x^2 - 7x + 10 = 0$. Find the quadratic equation whose roots are:

- (a) $x_1 + 1$ and $x_2 + 1$
 (b) $2x_1$ and $2x_2$
 (c) $\frac{1}{x_1}$ and $\frac{1}{x_2}$

10. **H** **The discriminant and Vieta.**

- (a) Show that $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$.
 (b) For the equation $ax^2 + bx + c = 0$, show that

$$(x_1 - x_2)^2 = \frac{b^2 - 4ac}{a^2} = \frac{D}{a^2}.$$

- (c) Use this to find $|x_1 - x_2|$ for the equation $x^2 - 5x + 3 = 0$ without finding the roots.

11. **M** Solve the equation

$$x^4 - 5x^2 + 4 = 0.$$

Hint: Let $y = x^2$.

12. **M** Let x_1, x_2, x_3 be the roots of the cubic equation

$$x^3 - 3x^2 + 2x + 4 = 0.$$

Find:

- (a) $x_1 + x_2 + x_3$ (c) $x_1x_2x_3$
 (b) $x_1x_2 + x_1x_3 + x_2x_3$ (d) $x_1^2 + x_2^2 + x_3^2$

13. **H** **Inequalities involving means.**

- (a) Prove that for any $a > 0$,

$$a + \frac{1}{a} \geq 2,$$

with equality only when $a = 1$.

- (b) Show that for any $a, b \geq 0$,

$$\frac{a+b}{2} \geq \sqrt{ab}.$$

The left-hand side is called the *arithmetic mean*, and the right-hand side is the *geometric mean*. This is the famous **AM–GM inequality**.

Hint: Start from $(\sqrt{a} - \sqrt{b})^2 \geq 0$.

14. **H** **Challenge: A recursive formula for powers.** Let x_1 and x_2 be the roots of $x^2 - 3x + 1 = 0$, and define $S_n = x_1^n + x_2^n$.

- (a) Find S_0, S_1 , and S_2 .
 (b) Prove that for all $n \geq 2$: $S_n = 3S_{n-1} - S_{n-2}$.
Hint: Since x_1 is a root, $x_1^2 = 3x_1 - 1$. Multiply both sides by x_1^{n-2} .
 (c) Use this recurrence to find S_5 .

15. **H** Challenge: When are the roots reciprocals? For what value(s) of k does the equation

$$kx^2 - (2k + 3)x + k = 0$$

have roots that are reciprocals of each other (i.e., $x_1x_2 = 1$)?

Quick Check Answers

Quick Check 1 (page 1):

1. Sum = 8, Product = 15
2. $x_1 + x_2 = -2$, $x_1x_2 = -3$
3. $\frac{4}{3}$

Quick Check 2 (page 2):

1. (a) Yes (b) No (c) Yes (d) No
2. 20
3. 95

Quick Check 3 (page 3):

1. (a) -2 (b) -3
2. + (positive)
3. 14