Math 6e: Homework 9

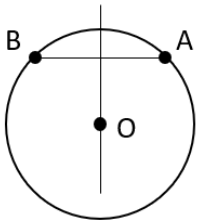
HW#9 is due November 19th; submit it to Google Classroom 15 minutes before class.

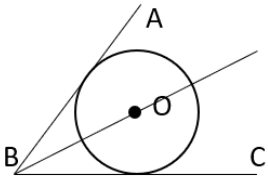
Please, write clearly which problem you are solving and show all steps of your solution.

Summary from the classwork

- **Operations** we can do using a ruler and a compass.
  1. Construct the midpoint of a given segment AB.
  2. Construct the perpendicular bisector of segment AB, i.e. a line that goes through the midpoint of AB and is perpendicular to AB. (also a bisector to a chord on a circle)
  3. Given a line l and a point A on l, construct a perpendicular to the line l through A.
  4. Given a line l and a point P outside of l, construct a perpendicular to the line l through P.
  5. Given an angle AOB, construct the angle bisector (i.e., a ray OM such that $\angle AOM \cong \angle BOM$)

- **Important constructions and definitions**
  - ***Center of a circle:*** If two points A, B are on a circle, then the center of this circle lies on the perpendicular bisector to AB (i.e., a line that goes through the midpoint of AB and is perpendicular to AB).
  - ***Circle inscribed in an angle*** (circle inside an angle). If a circle is inscribed in the angle ABC, then the center of this circle lies on the angle bisector.
  - ***Circle inscribed in a triangle*** (circle inside a triangle). Construct 1) two angle bisectors that define the center of the circle and 2) a $\perp$ line to one of the sides that passes through the center of the circle.
  - ***Circumscribed circle*** (circle around a triangle). Construct 1) two symmetry lines (bisectors) of two sides that define the center of the circle and 2) radius from the center of the circle to one of the triangle's vertices.





- **Solution format** for every problem (*what we need to include):*
  A) ***Give a recipe*** (procedure) for constructing the required figure using only a ruler and compass
  B) ***Analysis.*** Prove or explain why our recipe gives the correct answer

Homework questions

All construction problems should be done using only a ruler and a compass. Use one page per problem and draw large objects.

1. Given a circle (*draw a circle*), find its center. (Hint: construct 2 chords, then construct their bisectors)

2. Given a triangle ABC *(draw a triangle)*, construct a circle inscribed in the triangle. (Hint: construct angle bisectors)

3. Construct two concentric circles, where the circles have the same center and one has a larger radius. Draw a line that crosses both circles at consecutive points A, B, C, and D; the line should not pass through the centers. Prove that AB = CD.

4. Six grasshoppers sit on a road. Every minute, one grasshopper jumps 1 foot in one direction (along the road), and another grasshopper jumps 1 foot in the opposite direction. If initially the grasshoppers were at positions 1ft, 2ft, …, 6ft (measured from some point on the road), is it possible that after some time they will all gather at the same place on the road?