

## Math 6e: Homework 25

HW#25 is due April 22; submit to Google Classroom 15 minutes before class.

*Please, write clearly which problem you are solving and show all steps of your solution.*

### Arithmetic sequences

A sequence of numbers is an *arithmetic sequence* if the difference between consecutive terms is the same number, a common difference, let's call it  $d$ . For example, consider the sequence:

$$1, 5, 9, 13, 17, \dots$$

The first term in the sequence is:  $a_1 = 1$ , the second is  $a_2 = 5$ , and so on. The difference is  $d = 4$ . What is the  $n^{\text{th}}$  term? For example, what is  $a_{100}$

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = a_1 + 2d = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

...

$$\mathbf{a_n = a_1 + (n - 1)d}$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

### Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To prove this, we can write:

$$a_n = a_{n-1} + d$$

$$a_n = a_{n+1} - d$$

Add the two expressions above, and we have:  $2a_n = (a_{n-1} + d) + (a_{n+1} - d)$

$$2a_n = a_{n-1} + a_{n+1}$$

Dividing by 2 both sides:

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To find the common difference between 2 terms  $a_s$  and  $a_t$ :  $d = \frac{a_s - a_t}{s - t}$

### Sum of an arithmetic sequence

$$S = a_1 + a_2 + a_3 + \dots + a_n = n \times \frac{a_1 + a_n}{2}$$

To prove this, we write the sum in 2 ways, in increasing order, and in decreasing order:

$$S = a_1 + a_2 + a_3 + \dots + a_n$$

$$S = a_n + a_{n-1} + a_{n-2} + \dots + a_1$$

Adding up:

$$2S = (a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \dots$$

We notice that:

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$$

$$2S = (a_1 + a_n) \times n$$

$$S = \frac{(a_1 + a_n) \times n}{2}$$

## Homework questions

1. Write the first 5 terms of the arithmetic sequence for which  $a_3 = 7$  and  $d = 12$
2. What are the first 2 terms of the arithmetic sequence  $a_1, a_2, -9, -2, 5, \dots$
3. If for an arithmetic sequence  $a_{10} = 131$  and  $d = 12$ , what is  $a_1$ ? [Hint: use the equation for  $a_n = a_1 + (n - 1)d \dots$  and solve for  $a_1$ ]
4. If for an arithmetic sequence  $a_5 = 27$  and  $a_{27} = 60$ , find the first term and the common difference. [Hint: use an equation to find  $d$  first]
5. Find the common difference in an arithmetic sequence if the 9<sup>th</sup> term is 18 and the 11<sup>th</sup> term is 44.
6. Find the sum of the first 100 terms of an arithmetic sequence if  $a_1 = 10$  and  $a_{100} = 150$ .
7. Find the sum of all odd numbers from 1 to 2011.
8. Can you continue the following sequence:  $-2, 1, 6, 13, 22, \dots$   
[Hint: look at the differences of successive terms!] Write at least 3 more terms.

### Review from coordinate geometry:

9. Find the shortest distance from the origin  $(0, 0)$  to a line given by the equation  $y = -2x + 8$ . Start by sketching the graph of the line.
10. Compute the area of the rectangle  $ABCD$  if  $A$  is at  $(0, 0)$ ,  $B$  at  $(2, 3)$ , and  $D$  at  $(-6, 8)$ .  
[This can be done in more than one way.]