Math 6e: Homework 1

HW#1 is Due September 24; submit to Google classroom 15 minutes before the class time.

Review from Math 5

1. A boy had a bag of apples. He gave one-half of them to his parents, one-fifth to his brother, one-fourth to his sister, and he ate the last apple himself. How many apples did he initially have?

2. Simplify the following expressions

a) $x + 4(1 - x)$ b) $2 + 5x - 4(3 - x)$

c) $5(x - 1) - 3(2x + 1)$ d) $(2x + 5y)(3x + y + 2)$

Hint: Use the distributive property of multiplication: $a(b + c) = ab + ac$

Example: $(x + y)(z + d) = x(z + d) + y(z + d)$

3. Two secretaries, Barbara and Mary, need to type a 100-page document. Barbara can type it in 4 hours; Mary types slower, so it would take her 5 hours to do this. How fast can they type it together if they divide the work between the two of them most efficiently?

4. Simplify the following expression:

$$\frac{(x^2y^2) \cdot x^3}{x^2y^5}$$

5. Let $a = 2 \cdot 10^8$, $b = 10^5$. Compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$

6. If $a = 2^{-13}3^9$, $b = 2^{11}3^{-7}$ what is the value of ab? of a/b?

7. If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the length of the other leg?

8. Simplify: $\left(\sqrt{17}\right)^2, \left(\sqrt{13}\right)^4, \left(\sqrt{11}\right)^3, \sqrt{2^4 3^6}, \sqrt{2^4 3^5}$

9. Repeat each calculation by writing it again, and then write under each one which power rules were used

a) $2^2 \cdot (2^3)^5 = 2^2 \cdot 2^{3 \cdot 5} = 2^2 \cdot 2^{15} = 2^{2+15} = 2^{17}$

b) $\frac{2^2 \cdot 2^{-3}}{(2^{-2})^2 \cdot 2^4} = \frac{2^{2-3}}{2^{-2 \cdot 2} \cdot 2^4} = \frac{2^{-1}}{2^{-4} \cdot 2^4} = \frac{2^{-1}}{2^{-4+4}} = \frac{2^{-1}}{2^0} = \frac{2^{-1}}{1} = \frac{1}{2}$

c) $2^1 \cdot 2^2 \cdot 2^3 \cdot 2^4 = 2^{1+2+3+4} = 2^{10}$

d) $\frac{7^2 \cdot (7^{-2})^{-4}}{(7^2)^3 \cdot 7^{-13}} = \frac{7^2 \cdot 7^{-2 \cdot -4}}{7^{2 \cdot 3} \cdot 7^{-13}} = \frac{7^2 \cdot 7^8}{7^6 \cdot 7^{-13}} = \frac{7^{2+8}}{7^{6-13}} = \frac{7^{10}}{7^{-7}} = 7^{10-(-7)} = 7^{10+7} = 7^{17}$

e) $\frac{a^5 \cdot b^3}{a^3 \cdot b^5} = \frac{a^{5-3}}{b^{5-3}} = \frac{a^2}{b^2}$

f) $\frac{7a^2b^3}{14(a^3)^2b^{-3}} = \frac{a^2b^3}{2a^{3 \cdot 2}b^{-3}} = \frac{a^2b^{3-(-3)}}{2a^6} = \frac{a^2b^{3+3}}{2a^6} = \frac{b^6}{2a^{6-2}} = \frac{b^6}{2a^4}$

g) $\frac{8^2}{4^2} = \frac{(2^3)^2}{(2^2)^2} = \frac{2^{3 \cdot 2}}{2^{2 \cdot 2}} = \frac{2^6}{2^4} = 2^{6-4} = 2^2$

Power Rules

A) Multiplication $a^m \cdot a^n = a^{m+n}$

B) Raised to a power : $(a^m)^n = a^{m \cdot n}$

C) No power: $a = a^1$

D) Zeroth power: $a^0 = 1$

E) Switching numerator-denominator: $a^n = \frac{1}{a^{-n}}$ or $\frac{1}{a^n} = a^{-n}$

F) Division: $\frac{a^m}{a^n} = a^{m-n}$ if m>n or $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ if m<n

G) Change of base

Use powers of numbers to get to a smaller or larger base

Example: $32 = 2^5$ or $9 = 3^2$