Math 6e: Homework 19

HW#19 is due February 18; submit to Google Classroom 15 minutes before class.
Please, write clearly which problem you are solving and show all steps of your solution.

Sets: counting

- We use $|A|$ to denote the number of elements in a set A (if this set is finite). For example, if $A = \{a,b,c,\ldots,z\}$ is the set of all letters of the English alphabet, then $|A| = 26$.
- If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$
  For example, if there are 13 girls and 15 boys in the class, then the total is 28.
- If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Sets: product rule

- If we need to choose a pair of values, and there are a- ways to choose the first value and b- ways to choose the second, then there are ***ab*** ways to choose the pair.
  For example, a position on a chessboard is described by a letter and a number like *f4*; there are 8 possible choices for the letter, and 8 possible choices for the digit, so there are $8 \times 8 = 8^2 = 64$ possible positions.

- It works similarly for triples, quadruples, …
  For example, if we toss a coin, there are 2 possible outcomes: heads (H) or tails (T). If we toss a coin 4 times, the result can be written by a sequence of four letters, e.g. HTHH; since there are 2 possibilities for each of the letters, there are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible sequences.

Homework questions

1. Let set A contain the numbers 1, 2, 3 written in set notation as $A = [1, 2, 3] = \{x | 1 \leq x \leq 3\}$. Set B has elements called x that are all greater than or equal to 3, written in set notation as $B = \{x | x \geq 3\}$, and set C contains elements x that are all greater than or equal to 1.5, written in set notation as $C = \{x | x \leq 1.5\}$. Draw on a number line the following sets (one number line per set):

   (a) $\bar{A}$ (b) $\bar{B}$ (c) $\bar{C}$
   (d) $A \cap B$ (e) $A \cap C$
   (f) $A \cap (B \cup C)$ (g) $A \cap B \cap C$

2. For each of the 3 sets below (a), (b), and (c), do: 1) draw the number line and the elements on the line, and 2) describe its complement (everything not in the set) that completes to the "full set" that is + or - ∞ :
   (a) [0 , 2] **(b)** $(-\infty , 1] \cup [3 , \infty)$ **(c)** $(0 , 5) \cup (2 , \infty)$, where the notation means

   $[a , b] = \{x \mid a \leq x \leq b\}$ is the interval from a to b (including endpoints),
   $(a , b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
   $[a , \infty) = \{x \mid a \leq x\}$ is the half-line from a to infinity (including a),
   $(a , \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a).

3. Long ago, in some town, a phone number consisted of a letter followed by 3 digits (e.g., K651). How many possible phone numbers could there be? [Note that digits could be zero, i.e. X000 is allowed.]

4. If we roll 3 dice (one red, the other white, and the third black), how many possible combinations are there? How many combinations give the sum of values to be exactly 4?

5. (**Optional**): In this problem, we use $|A|$ to denote the number of elements in a finite set A. We know that for two sets A and B, we have $|A \cup B| = |A| + |B| - |A \cap B|$

   Can you come up with a similar rule for three sets: that is, write a formula for $|A \cup B \cup C|$ which uses $|A|, |B|, |C|, |A \cap B|, |A \cap C|, |B \cap C|$

6. In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that exactly one girl plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer, chess, neither, or both?