Math 6e: Homework 18

HW#18 is due on February 11; submit to Google Classroom 15 minutes before class.

Please, write clearly which problem you are solving and show all steps of your solution.

Sets

By the word *set*, we mean any collection of objects: numbers, letters, etc. Most of the sets that we consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g., {1, 2, 3}
- By giving some conditions, e.g., "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \ldots\ldots\}$, where dots stand for some condition (equation, inequality, etc.) involving x and where all x satisfy this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all x such that $x^2 > 2$".

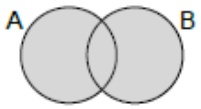
Other notations:

$x \in A$ means "x is in A", or "x is an element of A"

$x \notin A$ means "x is not in A"

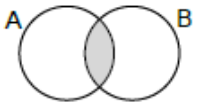
$A \cup B$: union of A and B. It consists of all elements that are in either A or B (or both):

$$A \cup B = \{x | x \in A \; OR \; x \in B\}$$



$A \cap B$: intersection of A and B. It consists of all elements that are in both A and B:

$$A \cap B = \{x | x \in A \; AND \; x \in B\}$$



$\bar{A}$: complement of A, i.e., the set of all elements which are not in A, $\bar{A} = \{x | x \notin A\}$

$A \subset S$, A is a subset of S, all elements of A are elements of S.

$A = \emptyset$, an empty set. A has no elements.

DeMorgan's laws: $\overline{A \cup B} = \bar{A} \cap \bar{B}$, $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Homework questions

1. If Al comes to a party, Betsy will not come. Al never comes to a party where Charley comes. And either Betsy or Charley (or both) will for sure come to the party.

Based on all of this, can you explain why it is impossible for Al to come to the party?

2. Let

A = set of all people who know French
B = set of all people who know German
C = set of all people who know Russian
Not that in each set, some people may speak more than one language.

Describe in words the following sets (which languages are spoken):

(a) $A \cap B$ (b) $A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \bar{A}$.

3. Let us take the usual deck of cards. As you know, there are 4 suits: hearts, diamonds, spades and clubs, 13 cards in each suit.

Make the following sets:

H = set of all hearts cards
Q = set of all queens
R = set of all red cards

Describe by formulas (such as $H \cap Q$) the following sets:

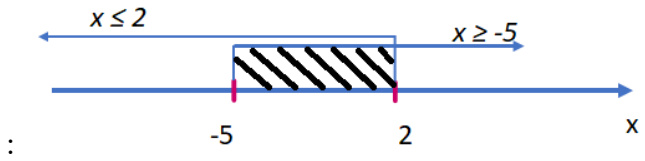
all red queens
all black cards
all cards that are either hearts or a queen
all cards other than red queens

How many cards are there in each set?

4. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French? Hint: draw a Venn diagram

5. Draw a number line. Shade the following sets on the number line:

(a) ***Example and solution:*** Set of all numbers x satisfying $x \leq 2$ **and** $x \geq -5$;



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(b) Set of all numbers x satisfying $x \leq 2$ **or** $x \geq -5$;
(c) Set of all numbers x satisfying $x \leq -5$ **or** $x \geq 2$.

6. (Optional) Use Venn diagrams to prove that the left and the right side of the second DeMorgan's law are equivalent: $\overline{A \cap B} = \bar{A} \cup \bar{B}$