

## Math 66: Homework 16

HW#16 is due January 28; submit it to Google Classroom 15 minutes before class.

*Please, write clearly which problem you are solving and show all steps of your solution.*

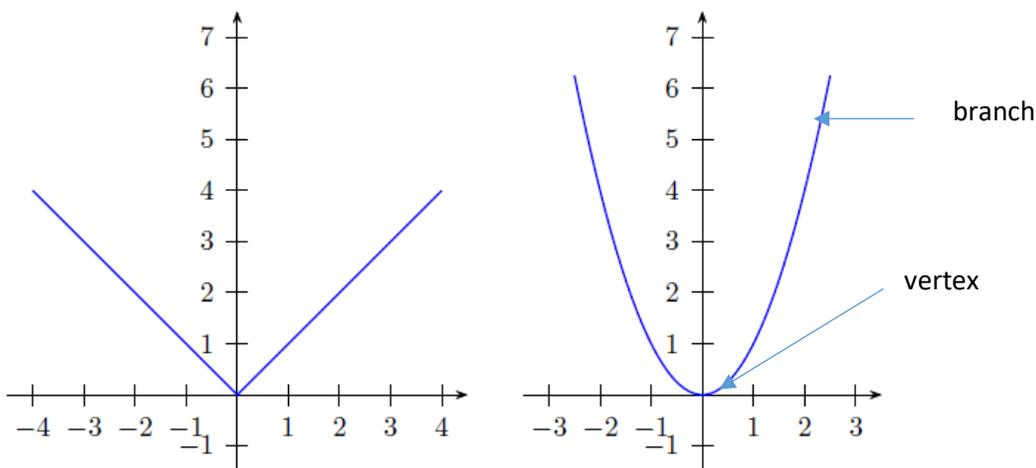
### Graphs

Generally, a graph of a linear function,  $y = f(x)$ , is a line in the  $x - y$  plane. If one has two graphs  $y = f(x)$  and  $y = g(x)$  one can find **intersection** points of corresponding graphs by solving the system of equations.

For example, the intersection point of two straight lines  $y = x + 2$  and  $y = -x$  is the point  $(-1, 1)$  as  $x = -1$  and  $y = 1$  satisfy both of these equations; that is the point  $(-1, 1)$  belongs simultaneously to both straight lines.

### Graphs of $y = |x|$ and $y = x^2$

The figures below show graphs of functions  $y = |x|$  and  $y = x^2$  (a quadratic function in powers of  $x$ ); the second graph is called a *parabola*.



The **standard form** of a parabola,  $y = ax^2 + bx + c$ , is hard to visualize and graph immediately.

In its **vertex form**, the parabola's coefficients  $a$ ,  $h$ , and  $k$  are directly related to the shape of the graph

$$y = a(x - h)^2 + k \quad (\text{vertex form}), \text{ where } h = -\frac{b}{2a} \text{ and } k = -\frac{b^2 - 4ac}{4a}.$$

The graph of a parabola with nonzero  $a, k, h$  coefficients, compared to  $y = x^2$ , is vertically stretched by a factor of  $a$  (if  $a < 0$ , this means flipping it upside down and then stretching by  $|a|$ ), and then its vertex is moved to point  $(h, k)$ . In particular, the branches go up if  $a > 0$  and down if  $a < 0$ .

You can **convert from standard to vertex form**. List the coefficients  $a, b, c$  from the standard form, then calculate  $h$  and  $k$  from the equations above, and after that rewrite the graph equation into its vertex form  $y = a(x - h)^2 + k$ . For example,  $y = x^2 + x$  can be converted into  $y = (x + \frac{1}{2})^2 - \frac{1}{4}$

The parabola either intersects  $y = 0$  ( $x - \text{axis}$ ) at two points, does not intersect it, or touches  $y = 0$  at a single point. These points of intersection are known as **roots**. Correspondingly, the quadratic equation has two roots, no roots, or one root, respectively. One can easily check that this corresponds to  $D > 0$ ,  $D < 0$  and  $D = 0$  respectively, where the **determinant**  $D = b^2 - 4ac$  is found using the quadratic equation in a standard form.

## Homework questions

*To draw a graph of an equation, choose a set of points  $x$  and find the corresponding  $y$  values. Draw the points on a graph and use a quadrille (square) paper. Connect with a smooth curve.*

1. Find the equation of the line that passes through the point (3,4) and has a slope +2. (Hint: you only need to find the intercept and write  $y = ax + b$ )
2. Find the equation of the line through the points (-2, 0) and (0, 2).
3. Sketch the graph of the functions:  $y = |x + 1|$  and  $y = -x + 0.25$ . How many solutions do you think the following equation has?

$$|x + 1| = -x + 0.25$$

Note: you are not asked to solve the equation – just answer how many solutions there are.

4. Find the intersection point of a line  $y = \frac{1}{4}x^2$  and a line  $y = 2x + 1$ . Sketch or draw the graphs. (Hint: construct a system of equations and solve.)
5. Sketch/draw graphs of the following functions (you may use Desmos for this question). **Then clearly describe the similarities and the differences between these graphs using complete sentences.**
  - a)  $x + y = 2$
  - b)  $y = |x - 5| + 1$
  - c)  $y = |x + 1| + |x - 2|$
  - d)  $y = |x + 1| + |x + 2| + |x + 3|$
6. Sketch/draw graphs of the following function:  $y = -x^2 + 4x - 3$ 
  - a) To sketch, convert the function from standard to vertex form and use your knowledge of what the coefficients  $a$ ,  $h$ , and  $k$  mean.
  - b) If you cannot convert to vertex form, select  $x$  values for a few points, then calculate the corresponding  $y$ -values as you would to graph any other function.
  - c) Does the graph intersect the  $x$ -axis (when in the parabola's equation  $y$  is set to 0)? The points of intersection are known as **roots**.
  - d) Does the number of roots correspond to the D-value? (\*calculating the determinant is optional)