

MATH 6. ASSIGNMENT 7: SETS

NOVEMBER 2, 2025

SETS

By word *set*, we mean any collection of objects: numbers, letters,... Most of the sets we will consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

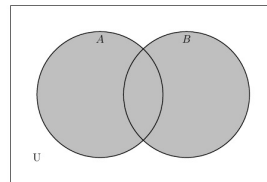
- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. “set of all numbers satisfying equation $x^2 > 2$ ”. In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x , denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means “set of all x such that $x^2 > 2$ ”.

Other notation:

$x \in A$ means “ x is in A ”, or “ x is an element of A ”

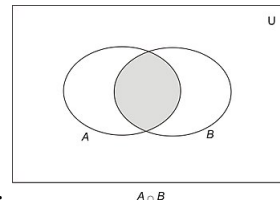
$x \notin A$ means “ x is not in A ”

$A \cup B$: union of A and B . It consists of all elements which are in either A or B (or both):



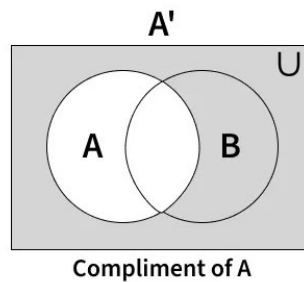
$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$

$A \cap B$: intersection of A and B . It consists of all elements which are in both A and B :

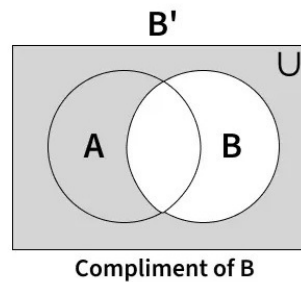


$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$

$\bar{A}$: complement of A , i.e. the set of all elements which are not in A :



$$\bar{A} = \{x \mid x \notin A\}.$$



$$\bar{B} = \{x \mid x \notin B\}.$$

HOMEWORK

1. Consider the operation **NAND** which is just the opposite of **AND**. See Classwork 6 for hints. Using only the component **NAND**, see if you can create circuits equivalent to **AND**, **OR**, and **NOT**.
2. Using only **AND**, **NOT**, and **OR**, produce a **three-input AND circuit**, i.e., the output is **F** unless all three inputs are **1** or **T**. (You do not have to use all three circuit elements.)
3. Using only **AND**, **NOT**, and **OR**, produce a **three-input OR circuit**, i.e., the output is **1** or **T** if any of the inputs is **1** or **T**.
4. If **Al** comes to a party, **Betsy** will not come. **Al** never comes to a party where **Charley** comes. And either **Betsy** or **Charley** (or both) will certainly come to the party.
Based on all of this, can you explain why it is impossible that **Al** comes to the party?

5. Let

A =set of all people who know French

B =set of all people who know German

C =set of all people who know Russian

Describe in words the following sets:

- (a) $A \cap B$ (b) $A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \bar{A}$.

6. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.

Denote:

H =set of all hearts cards

Q =set of all queens

R =set of all red cards

Describe by formulas (such as $H \cap Q$) the following sets:

all red queens

all black cards

all cards that are either hearts or a queen

all cards other than red queens

How many cards are there in each set?

7. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?
8. Simplify the following expressions.

(a) $\frac{6^6 \cdot 2^2}{3^9 \cdot 2^{17}} =$

(b) $\frac{(15^3)^3 \cdot 3^9}{5^{15}} =$

(c) $(7^7 \cdot 7^3)^2 =$

(d) $\frac{2^{-2}}{2^{-12}} =$