

MATH 6

HOMEWORK 5: TRUTH TABLES AND LOGIC LAWS

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TRUTH TABLES

Logical variables: take value True (T) or False (F).

Basic logic operations:

NOT (NOT A): true if A is false, and false if A is true.

A	NOT A
T	F
F	T

AND (A AND B): true if both A, B are true, and false otherwise

OR (A OR B) **inclusive or**: true if at least one of A, B is true, and false otherwise.

XOR (A XOR B) **exclusive or**: true if different, false if the same.

IF (as in “if A , then B ; written $A \implies B$): if A is false, automatically true; if A is true, it is true only when B is true

Logic operations can be combined, e.g. $(A$ OR $B)$ AND C .

Truth tables: *

A	B	A XOR B
T	T	F
T	F	T
F	T	T
F	F	F

A	B	A AND B
T	T	T
T	F	F
F	T	F
F	F	F

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Truth tables are useful in solving the problems about knights and knaves. Here is a typical problem: on the island of knights and knaves you meet two inhabitants, Zed and Alice. Zed tells you, ‘I am a knight or Alice is a knave.’ Alice tells you, ‘Of Zed and I, exactly one is a knight.’ We could solve it by making the following table:

Zed	Alice	Z is a knight or A is a knave	Of Z and A, exactly one is a knight
knight	knight	T	F
knight	knave	T	T
knave	knight	F	T
knave	knave	T	F

LOGIC LAWS

We can combine logic operations, creating more complicated expressions such as A AND $(B$ OR $C)$. As in arithmetic, these operations satisfy some laws: for example A OR B **is the same as** B OR A . Here are two other laws:

NOT(A AND B) is the same as (NOT A) OR (NOT B)

$A \implies B$ is the same as (NOT B) $\implies$ (NOT A)

Truth tables provide the easiest way to prove complicated logical rules: if we want to prove that two formulas are equivalent (i.e., always give the same answer), make a truth table for each of them, and if the tables coincide, they are equivalent.

- On the island next to the island of knights and knaves there are 3 kinds of people:
knights, who always tell the truth
knaves, who always lie
normal people, who sometimes lie and sometimes tell the truth
On that island, you meet 3 people, A, B, and C, one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements:
A: I am normal
B: That is true
C: I am not normal
What are A, B, and C?
- Check whether $A \implies B$ and $B \implies A$ are equivalent, by writing the truth table for each of them.
- Check that $A \implies B$ is equivalent to $(\text{NOT } A) \text{ OR } B$ (thus, “if you do not clean up your room, you will be punished” and “clean up your room, or you will be punished” are the same).
- A teacher tells the student “If you do not take the final exam, you get an F”. Does it mean that
 - If the student does take the final exam, he will not get an F
 - If the student does not get an F, it means he must have taken the final exam.
- Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)?
 - $(A \text{ OR } B) \text{ AND } (A \text{ OR } C)$
 - $A \text{ OR } (B \text{ AND } C)$.
- Define a new logical operation, XOR (exclusive or) as follows: $A \text{ XOR } B$ is true if exactly one of A, B is true, and false otherwise.
 - Write the truth table for $A \text{ XOR } B$.
 - Can you express XOR using only AND, OR, and NOT (that is, write a formula equivalent to $A \text{ XOR } B$ using only AND, OR, and NOT)? **Hint: create truth table with columns: $A, B, \bar{A}, \bar{B}, A\bar{B}, \bar{A}B$, and think which logical operations can give you XOR output.**
- Write truth tables for formulas $A \text{ AND } (B \text{ OR } C)$ and $(A \text{ AND } B) \text{ OR } C$ (hint: there will be 8 rows in the table). Are these formulas equivalent (i.e., do they always give the same answer)?
 - The waiter in a restaurant tells you: “our fixed price dinner includes soup and appetizer or salad.” Denoting
 A = your dinner will include soup
 B = your dinner will include appetizer
 C = your dinner will include salad
what would be the correct way to write his statement using letters A, B, C and logical operations AND, OR?
- You probably know Lewis Carroll as the author of *Alice in Wonderland* and other books. What you might not know is that he was also a mathematician very much interested in logic, and had invented a number of logic puzzles. Here is one of them:
You are given 3 statements.
 - All babies are illogical.
 - Nobody is despised who can manage a crocodile.
 - Illogical persons are despised.
Can you guess what would be the natural conclusion from these 3 statements?