CLASSWORK 1,

September 14, 2025 Review 1

Power properties:

$$(ab)^n = \underbrace{ab \cdot ab \cdots ab}_{n\ times} = \underbrace{a \cdot a \cdots a}_{n\ times} \cdot \underbrace{b \cdot b \cdots b}_{n\ times} = a^n b^n$$

$$a^m a^n = \underbrace{a \cdot a \cdots a}_{m\ times} \cdot \underbrace{a \cdot a \cdots a}_{n\ times} \cdot = \underbrace{a \cdot a \cdots a}_{m+n\ times} = a^{m+n}$$

$$\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdots a}^{m\ times}}{\underbrace{a \cdot a \cdots a}_{n\ times}} = \underbrace{a \cdot a \cdots a}_{m-n\ times} = a^{m-n} \qquad if\ m > n$$

$$\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdots a}^{m\ times}}{\underbrace{a \cdot a \cdots a}_{n\ times}} = \frac{1}{\underbrace{a \cdot a \cdots a}_{n-m\ times}} = \frac{1}{a^{n-m}} \qquad if\ m < n$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-3} = \frac{1}{a \cdot a \cdot a} = \frac{1}{a^3}$$

$$a^{-n} = \frac{1}{a^n}$$

Binary numbers

n	0	1	2	3	4	5	6	7	8	9
$\mathbf{2^n}$	1	2	4	8	16	32	64	128	256	512

Numbers in decimal notation can be presented like this

$$351 = 3 \cdot 100 + 5 \cdot 10 + 1 \cdot 1$$

Here how we can find a binary representation of a decimal number

351=**256** + 95 = **256 + 64** + 31 **= 256 + 64 + 16** + 15 **= 256 + 64 + 16 + 8** + 7 = **256 + 64 + 16 + 8 + 4 + 2 + 1**

$351 = 1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 101011111b$

HOMEWORK 1, Review

Please use quadrille paper to write your homework. Quadrille is the paper with squares, not with lines. Please write enough details about how you solved the problem. Pictures are very welcome.

1. Simplify the following and show the answer in the exponent form

a) $\frac{3^7 \cdot 2^7}{2^3 \cdot 2^4} =$ b) $\frac{6^5 \cdot 2^4}{3^5 \cdot 2^2} =$ c) $\frac{7^9 \cdot 2^5}{7^2 \cdot 2^4} =$

d) $\frac{11^4}{11^2 \cdot 5^2 \cdot 5^3} =$ e) $7^4 \cdot 11^2 \cdot 11^{-5} \cdot 7^2 =$ f) $\frac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} =$

g) $\frac{42^2}{6^2} =$ h) $\frac{3^5 \cdot 3^{-5}}{3^9} =$ i) $\frac{x^2 \cdot y^2 \cdot x^{-3}}{x^2} =$

2. Let $a = 2 \cdot 10^8$, $b = 10^5$, compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$.

3. Convert decimal numbers to binary:
   9, 12, 24, 38, 45
4. Convert the following binary numbers to decimal:
   101, 1001, 10110, 11011, 10101
5. You have scales (with two platforms), a 1-gram weight and a large bag of sugar. What would be the fastest way to measure exactly 8 grams of sugar? Exactly 128 grams? Exactly 100 grams?
6. Solve the following equations:
   a) $2(x - 1) = \frac{2}{3}(x + 5)$
   b) $|x + 11| = 2$
   c) $\frac{x-2}{x-1} = 3$
7. A swimming pool can be filled by an inlet pipe in 12 hours and emptied by an outlet pipe in 15 hours. One day the pool is empty, and the owner opens the inlet pipe to fill it. However, he forgets to close the outlet pipe. How long will it take to fill the pool?
8. Find the following square roots. If you cannot find the exact number exactly, simplify the best you can
   a. $\sqrt{16}$
   b. $\sqrt{81}$
   c. $\sqrt{10{,}000}$
   d. $\sqrt{10^8}$
   e. $\sqrt{50}$