MATH 6: HOMEWORK 17

Arithmetic sequence

A sequence of numbers is an arithmetic sequence if the difference between consecutive terms is the same number, the **common difference**, let's call it d.
For example, let's consider the sequence:$1, 5, 9, 13, 17, ...$
The first term in the sequence is $a_1 = 1$, the second is $a_2 = 5$, and so on. The difference is $d = 4$.
What is the n^{th} term? For example what is a_{100}?

$a_1 = 1$

$a_2 = a_1 + d = 1 + 4 = 5$

$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$

$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$

....

$a_n = a_1 + (n - 1)d$

So $a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$

Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To prove this we can write:

$$a_n = a_{n-1} + d$$
$$a_n = a_{n+1} - d$$

Add them up and we have $2a_n = a_{n-1} + a_{n+1}$, divide by 2 and we get $a_n = \frac{a_{n-1} + a_{n+1}}{2}$
To find the common difference between 2 terms a_s and a_t

$$d = \frac{a_s - a_t}{s - t}.$$

Sum of an arithmetic sequence

$S = a_1 + a_2 + a_3 + ... + a_n = n \times \frac{a_1 + a_n}{2}$
To prove this, we write the sum in 2 ways, in increasing and decreasing order:
$S = a_1 + a_2 + a_3 + ... + a_n$
$S = a_n + a_{n-1} + a_{n-2} + ... + a1$
Adding up, we notice that $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = ...$
$2S = (a_1 + a_n) \times n$ and dividing by 2 gives us $S = n \times \frac{a_1 + a_n}{2}$.

Homework Problems

1. Write the first 5 terms of an arithmetic sequence if $a_1 = 7$ and $d = 2$.

2. What are the first 2 terms for the sequence $a_1, a_2, -9, -2, 5, ...$

3. $a_{10} = 131$ and $d = 12$. What is a_1?

4. $a_5 = 27$ and $a_{27} = 60$. Find the first term and the common difference.

5. Find the common difference in an arithmetic sequence if the 9^{th} term is 18 and the 11^{th} term is 44.

6. Find the sum of the first 100 terms if $a_1 = 10$ and $a_{100} = 150$.

7. Find the sum of all odd numbers from 1 to 2011.

***8.** Can you continue the following sequence: -2, 1, 6, 13, 22, ... [Hint: look at the differences of successive terms!]

9. Find the shortest distance from the origin $(0, 0)$ to the line given by the equation $y = -2x + 8$.

10. Compute the area of the rectangle $ABCD$ if A is at $(0, 0)$, B at $(2, 3)$, and D at $(-6, 8)$. [It can be done in more than one way.]