MATH 6: ASSIGNMENT 10
DECEMBER 7, 2025

Factorials and permutations

If we are choosing k objects from a collection of n objects so that

a) order matters and

b) no repetitions allowed (i.e. you can't pick up the same object twice),

then there are

$$P = n(n-1)(n-2)\dots(n-k+1) \qquad (k\ factors)$$

ways to do it.

If we take k = n, it means that we are selecting one by one all n objects — so this gives the number of possible ways to order n objects:

$$n! = n(n-1)\dots 2 \cdot 1$$

(reads n factorial).

For example: there are 52! ways to mix the cards in the usual card deck.

Note that the number n! grow very fast:

$2! = 2$
$3! = 6$
$4! = 4 \cdot 3 \cdot 2 = 24$
$5! = 120$
$6! = 620$

In all the problems that ask you to compute something, it suffices to write an expression for the answer, e.g., $\frac{1}{2^{11}}$; it is not necessary to perform the multiplication.

Algebraic Identities, we have been deriving pretty much every other class.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

HOMEWORK

1. Do practice sheet.
2. About 1/6 of Americans have blue eyes. If we choose 10 people at random, what is the probability that all of them have blue eyes? That none has blue eyes? that at least one has blue eyes?

3. You have 6 books on the shelf.
   a) In how many ways you can arrange them if the books are different?
   b) In how many ways can you arrange them if all the books are identical?
   c) In how many ways can you arrange them if 5 books are identical?
   d) In how many ways can you arrange them if you have 3 books of one kind and 3 books of another kind?

4. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?
5. How many ways are there to split a class of 6 boys and 4 girls into two soccer teams fairly? (To keep the teams fair, each team must have 3 boys and 2 girls).

6. A puzzle consists of 9 small square pieces which must be put together to form a 3 x 3 square so that the pattern matches (this kind of puzzle is quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination a second, how long would it take you to try them all?

7. Simplify:

(a) $\left(\frac{5a^2b^5}{4a^3b^3}\right)^3 =$ (b) $\left(2z^2 \cdot 3z^3 \cdot z\right)^2 =$ (c) $\frac{(-ab)^8}{(ab)^2} =$

(d) $\left(\frac{3ab^3}{15b}\right)^2 \cdot \frac{75c}{a^2b^6} =$ (e) $\left(\frac{3a^5b^2}{21ab}\right)^2 \cdot \frac{7^4}{a^{16}b^2} =$ f) $\frac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} =$

8. Open parentheses, simplify, use the knowledge of algebraic identities:
   a. $(2x - \frac{1}{2x})^2 =$
   b. $(\sqrt{a} + \frac{1}{2\sqrt{a}})^2 =$