

Math 6c, homework 19.



Algebraic identities:

1. Square of a sum:

$$(a + b)^2 = a^2 + 2ab + b^2$$

2. Square of a difference:

$$(a - b)^2 = a^2 - 2ab + b^2$$

3. Difference of squares:

$$a^2 - b^2 = (a + b)(a - b)$$

4. Cube of a sum:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

5. Cube of a difference:

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

6. Sum of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

7. Difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

8. Square of three terms:

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

1. Write as polynomials, using the algebraic identities (or you can just multiply polynomials):

a. $\left(x + \frac{1}{3}\right)^2$; b. $\left(\frac{3}{4}x - \frac{1}{5}y\right)^2$; c. $(2c + 3)^2 - (2c + 3)(2c - 3)$;

d. $\left(-1\frac{1}{2}m^2 + \frac{2}{3}k\right)^2$; e. $(m + 2n)(m^2 - 2mn + 4n^2)$; f. $(c + 1)^3$;

2. Factorize, using the algebraic identities:

a. $4x^2 - 4x + 1$; b. $h^3 - 27$; c. $9a^2 - 4$; d. $\frac{1}{4}m^2 - 16n^2$;

3. Simplify expressions:

a. $(x + 3)^2 + 3(x - 2)^2$; b. $(x + 1)(x^2 - x + 1) - (x^2 - 1)x$;

4. Show that

$$(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2) = a^6 - b^6$$