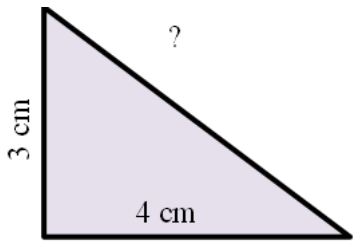
Math 6c, homework 14.

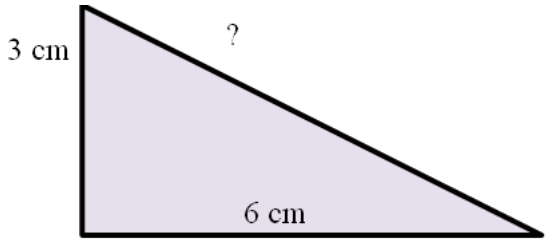
1. Without using calculator compare:

$3 \ldots \sqrt{11}$ $\quad 11 \ldots \sqrt{110}$ $\quad 22 \ldots \sqrt{484}$

$5 \ldots \sqrt{20}$ $\quad 17 \ldots \sqrt{299}$ $\quad 35 \ldots \sqrt{1215}$

2. For the right triangles below find the missing side:





3. Prove that the value of the following expressions is a rational number.

Example:

$(\sqrt{3}-1)(\sqrt{3}+1) = \sqrt{3}\cdot\sqrt{3}+\sqrt{3}\cdot 1-1\cdot\sqrt{3}-1 = \sqrt{3}\cdot\sqrt{3}-1 = (\sqrt{3})^2-1 = 3-1=2$

a. $(\sqrt{2}-1)(\sqrt{2}+1)$; $\quad$ *b.* $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$

d. $(\sqrt{2}+1)^2+(\sqrt{2}-1)^2$ $\quad$ *d.* $(\sqrt{2}+1)^2+(\sqrt{2}-1)^2$

e. $(\sqrt{7}-2)^2+4\sqrt{7}$

4. Evaluate:

a. $\dfrac{-9^{-2}\cdot(-3^{-2})^3}{3^{-8}}$; $\quad$ *b.* $\dfrac{25^{-7}\cdot(5^2)^3}{125^{-3}}$; $\quad$ *c.* $\dfrac{(-2^{-15})\cdot(-2^2)^3}{2^{-9}}$; $\quad$ *d.* $\dfrac{(-7)^{-4}}{14^{-3}}$;

5. Which exponent should replace * so that the equality is true:

$$\frac{a^{*}\cdot(a^4)^3}{a^{-17}} = \frac{1}{a^2}$$

6. Find the permissible (allowed) values of the variable in the fraction.

$\dfrac{5}{x-1}$; $\quad \dfrac{3}{a-11}$; $\quad \dfrac{-7}{b+3}$; $\quad \dfrac{2a}{(a-5)(a+3)}$; $\quad \dfrac{3-x}{x^2-4}$; $\quad \dfrac{1}{(x^2-9)(x^2+25)}$;

7. Write the following numbers in scientific notation;

Example:
$231457 = 2.31457 \cdot 10^5 \approx 2.31 \cdot 10^5$;
$327892312 = 3.27892312 \cdot 10^8 \approx 3.28 \cdot 10^8$;
$0.00312734 = 3.12734 \cdot 10^{-3} \approx 3.13 \cdot 10^{-3}$

$a.\, 3412893;$ $b.\ 56231452;$ $0.00000342567;$ 0.0003421

8. a. Check the following equalities:

$$\frac{1}{2}-\frac{1}{3}=\frac{1}{2\cdot 3};\qquad \frac{1}{3}-\frac{1}{4}=\frac{1}{3\cdot 4};\qquad \frac{1}{4}-\frac{1}{5}=\frac{1}{4\cdot 5};\qquad \frac{1}{5}-\frac{1}{6}=\frac{1}{5\cdot 6};$$

b. Continue the chain of similar equalities. Write the algebraic expression for the pattern.
c. Evaluate:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{99\cdot 100}$$

d. Use the previous conclusions to simplify the following expressions:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)};$$

9. Show, that

$$n! \le n^n$$