Math 6c. Classwork 5.

Can non-decimal place-value system be created? For example, with base 5?
Let see, how we can create this kind of system (we use our normal digits).

Num_{10}	1	2	3	4	5	6	7	8	9	10
Num_5	1	2	3	4	10	11	12	13	14	20

11	12	13	14	15	16	17	18	19	20
21	22	23	24	30	31	32	33	34	40

21	22	23	24	25	26	27	28	29	30
41	42	43	44	100	101	102	103	104	105

We only have 5 digits (0, 1, 2, 3, 4), and 4 first "natural" numbers in such system will be represented as one-digit numbers. Number 5 then should be shown as a 2-digit number, with fist digit 1 (place - value equal to 5^1) and 0 of "units". Any number is now written in the form

$$\dots + 5^3 \cdot n + 5^2 \cdot m + 5^1 \cdot k + 5^0 \cdot p, \qquad n, m, k, p \quad \text{are} \quad 0, 1, 2, 3, 4$$

Let's write number 782 in 5-based system:

The highest power of 5 which is smaller than 782 is $5^4 = 625$, taken once, therefore

$$782 = 625 + 157$$

The highest power of 5, which is smaller than 157 is $5^3 = 125$, taken once:

$$782 = 625 + 157 = 625 + 125 + 32$$

The highest power of 5, which is smaller than 32 is $5^2 = 25$, taken once:

$$782 = 625 + 157 = 625 + 125 + 25 + 7 = 625 + 125 + 25 + 5 + 2$$

$$782 = 5^4 + 5^3 + 5^2 + 5^1 + 5^0 \cdot 2 = 11112_5$$

Another way to write a number in 5-based system:

$782 \div 5 = 156\, remainder\ \mathbf{2}$

$156 \div 5 = 31\, remainder\ \mathbf{1}$

$31 \div 5 = 6\, remainder\ \mathbf{1}$

$6 \div 5 = 1\, remainder\ \mathbf{1}$

$1 \div 5 = 0\, remainder\ \mathbf{1}$

If these remainders are read from the bottom up, the same result will appear. Can you explain why? This is the easiest technical way to transfer number from decimal to n-based system, but to understand why the answer is right one, we have to learn the concept.

Another example:

Write the number 893 in 4-base system.

The highest power of 4, which is smaller than 893 is $4^4 = 256$, taken three times:

$$893 = 256 \cdot 3 + 125$$

The highest power of 4, which is smaller than 125 is $4^3 = 64$, taken one time

$$893 = 256 \cdot 3 + 64 + 61$$

The highest power of 4, which is smaller than 61 is $4^2 = 16$, taken three times:

$$893 = 256 \cdot 3 + 64 + 16 \cdot 3 + 13 = 256 \cdot 3 + 64 + 16 \cdot 3 + 4^1 \cdot 3 + 1$$

$$893 = 4^4 \cdot 3 + 4^3 + 4^2 \cdot 3 + 4^1 \cdot 3 + 4^0 \cdot 1 = 31331_4$$

$893 : 4 = 223\, remainder\ \mathbf{1}$

$223 : 4 = 55\, remainder\ \mathbf{3}$

$55 : 4 = 13\, remainder\ \mathbf{3}$

$13 : 4 = 3\, remainder\ \mathbf{1}$

$3 : 4 = 0\, remainder\ \mathbf{3}$

From 4-based to decimal:

Write 2132_4

$2132_4 = 4^3 \cdot 2 + 4^2 \cdot 1 + 4^1 \cdot 3 + 4^0 \cdot 2 = 64 \cdot 2 + 16 \cdot 1 + 4 \cdot 3 + 2 = 158$

From 5-based to decimal:

$$3423_5 = 5^3 \cdot 3 + 5^2 \cdot 4 + 5^1 \cdot 2 + 5^0 \cdot 3 = 125 \cdot 3 + 25 \cdot 4 + 5 \cdot 2 + 3 = 375 + 100 + 10 + 3 = 488$$

Exercises:

1. Write the numbers 234_6 and 403_6 written in the 6-based place-value system (small number 6 shows that the number is not in decimal, but in 6-based system) in decimal system.

2. Write 673, 753 in base-6 system.

3. Do the addition:

$$2134_5 + 3141_5; \qquad 21112_4 + 3211_4;$$

4. Do the subtraction:

$$4334_5 - 3441_5; \qquad 31212_4 - 12231_4;$$

5. Binary to decimal:

$$100001_2, \qquad 110011_2$$

6. Decimal to binary:

$$132, \qquad 111$$

7. Do the addition and subtraction:

$$1101101_2 + 111101_2; \qquad 11100111_2 - 110011_2;$$

8. Do the multiplication:

$$101_2 \cdot 110_2; \qquad 12_4 \cdot 23_4$$

9. There is a balance scale with two pans and one weight of each of the following masses: 1 g, 3 g, 9 g, 27 g, and 81 g. How can you balance a 61 g mass placed on one pan of the scale?

10. Robert thought of a number not less than 1 and not more than 1000. Julia is allowed to ask only such questions to which Robert can answer “yes” or “no” (Robert always tells the truth). Can Julia determine the hidden number in 10 questions?

11. There is a bag of sugar, a scale and a weight of 1 g. Is it possible to measure 1 kg of sugar in 10 weights?

12. How can 127 one-dollar bills be distributed among seven wallets so that any amount from 1 to 127 rubles can be given without opening the wallets?

13. Write a 4-digit number, each digit 1 more than the previous digit (like 2345). Then write the 4-digit number with the same digits but in opposite order (like 5432). Subtract the smaller number from the greater one. Do it two more times using different digits. What did you notice? Can you explain it?