Classwork 23-2.

Equations with absolute values.

Absolute value of a number;

$$\begin{cases} |m| = m, if\ m \geq 0 \\ |m| = -m, if\ m < 0 \end{cases}$$

The simplest equation with absolute value is

$$|x| = k$$

where k is a number.

If k is negative, equation has no solutions, since the absolute value of any number cannot be negative. If k is nonnegative, both k and $-k$ are solutions (only 0, if $k = 0$).

Examples:

$|y| = 7.5, \quad y = 7.5, -7.5$

$|x| = -7.5, \quad no\ solutions.$

The simple linear equation with absolute value has the general form

$$|ax + b| = k$$

where xxx is the unknown, and a, b, and k are real numbers, with $k \geq 0$.

Otherwise, the equation has no solutions.

The expression inside the absolute value can be equal to either k or $-k$.

The equation

is equivalent to two equations

$$\begin{tikzcd} & {|ax + b| = k} \arrow[dl] \arrow[dr] & \\ {ax + b = k} & & {ax + b = -k} \end{tikzcd}$$

$$ax = k - b \qquad\qquad ax = -k - b$$

$$x = \frac{k - b}{a} \qquad\qquad x = \frac{-k - b}{a}$$

Examples:

$$|3x - 2| = 12$$

$$3x - 2 = 12 \qquad\qquad 3x - 2 = -12$$

$$3x = 12 + 2 \qquad\qquad 3x = -12 + 2$$

$$x = \frac{14}{3} = 4\frac{2}{3} \qquad\qquad x = \frac{-10}{3} = -3\frac{1}{3}$$

$$|-2x+8| = 14$$

$$-2x+8 = 14 \qquad\qquad -2x+8 = -14$$

$$-2x = 14-8 \qquad\qquad -2x = -14-8$$

$$x = \frac{6}{-2} = -3 \qquad\qquad x = \frac{-22}{-2} = 11$$

Chek the solutions for these two equations.

Equations which can be written in the general form

$$|ax+b| = cx+d$$

where x is unknown, and a, b, c, and d are real numbers, can be split into two equations:

$$ax+b = cx+d \qquad and \qquad ax+b = -(cx+d)$$

But only those value of x that make $cx+d \geq 0$ are solutions of the original equation.

Example:

$$|x-3| = 2x+1$$

$$x-3 = 2x+1 \qquad\qquad x-3 = -(2x+1)$$

$$x = -3-1 \qquad\qquad x-3 = -2x-1$$

$$x = -4 \qquad\qquad -3x = -3+1 = -2$$

$$x = -4 \qquad\qquad x = \frac{2}{3}$$

For $x = -4$:

$$2 \cdot (-4) + 1 = -7 < 0$$

For $x = \frac{2}{3}$:

$$2 \cdot \frac{2}{3} + 1 = \frac{4}{3} + 1 = 2\frac{1}{2} > 0$$

Solution $x = \frac{2}{3}$.

Another type of equations with absolute value can be written as:

$$|ax+b| = |cx+d|.$$

where x is the unknown, and a, b, c, and d are real numbers. By the definition of absolute value, the expressions $ax+b$ and $cx+d$ are either equal or have opposite signs; therefore, it is sufficient to consider two cases: $ax+b = cx+d$ or $ax+b = -(cx+d)$.

Example;

$$|2x-5| = |3x+6|$$

We only need to consider

$$\begin{array}{ccc} 2x-5=3x+6 & and & 2x-5=-(3x+6) \\ 3x-2x=-5-6 & & 2x-5=-3x-6 \\ x=-11 & & 3x+2x=-6+5 \\ & & x=-\frac{1}{5} \end{array}$$

Another possible pair of equations,

$$-(2x-5)=3x+6 \qquad and \qquad -(2x-5)=-(3x+6)$$

is equivalent to the first pair.

More examples:

1.

$\big||3-2x|-12\big|=10$

$$\begin{array}{ll} |3-2x|-12=10 & |3-2x|-12=-10 \\ |3-2x|=22 & |3-2x|=2 \end{array}$$

$$\begin{array}{llll} 3-2x=22 & 3-2x=-22 & 3-2x=2 & 3-2x=-2 \\ -2x=22-3 & -2x=-22-3 & -2x=2-3 & -2x=-2-3 \\ -2x=19 & -2x=-25 & -2x=-1 & -2x=-5 \\ x=-9.5 & x=12.5 & x=0.5 & x=2.5 \end{array}$$

2.

$\big||3+5x|-4\big|=12$

$$\begin{array}{ll} |3+5x|-4=12 & |3+5x|-4=-12 \\ |3+5x|=16 & |3+5x|=-16 \end{array}$$

$$\begin{array}{lll} 3+5x=16 & 3+5x=-16 & no\ solutions \\ 5x=16-3 & 5x=-16-3 & \\ 5x=13 & 5x=-19 & \\ x=\frac{13}{5} & x=-\frac{19}{5} & \end{array}$$

Intervals.

Let there be a coordinate axis x and two real numbers a and b satisfying the inequality a < b. The numbers a and b can be considered as coordinates of two distinct points on the x-axis, which we have also agreed to call points a and b.

Thus, the **segment [*a, b*]** (**closed interval**) is the set of all real numbers x satisfying the double inequality:

$$a \leq x \leq b$$

Points a and b are called the endpoints of the segment [*a, b*]. The endpoints of the segment [*a, b*] belong to this segment.

If both endpoints are excluded from the segment [*a, b*], we obtain a set of points denoted by (*a, b*) and called the **open interval** from a to b.

Thus, the **interval (*a, b*)** is the set of all real numbers *x* satisfying the double inequality:

$$a < x < b$$