Classwork 22.

Completing the full square.

Polynomial of a second degree can be represented as

$$ax^2 + bx + c$$

where a, b and c are numbers (can be positive or negative), and x is a variable.

We already know that the square of the sum:

$$(m+n)^2 = m^2 + 2mn + n^2$$

The formula for polynomial of the second degree looks similar to the sum of the square.

First, let's to factor out a from first two terms of expression. By doing this we will not change the value of the expression.

$$a \cdot \left(x^2 + \frac{b}{a}x\right) + c$$

For the next step we will rewrite the expression as

$$a \cdot \left(x^2 + 2 \cdot \frac{1}{2}\frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$

Here I added and subtracted the square of the half of the $\frac{b}{a}$. Now the first three terms of the polynomial look like a square of the sum:

$$a \cdot \left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c = a \cdot \left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$$

And finally

$$a \cdot \left(x + \frac{b}{2a}\right)^2 - a\frac{b^2}{4a^2} + c = a \cdot \left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

Let study the quadratic function.

Plot functions:

$a.$ $f(x) = x^2;$ $y = 5x^2;$ $g(x) = 0.5x^2;$ $y = -2x^2;$

$b.$ $f(x) = x^2 + 5;$ $y = 5x^2 - 5;$ $g(x) = -0.5x^2 + 5;$

$c.$ $f(x) = (x+2)^2;$ $y = (5x-2)^2;$ $g(x) = (-0.5x - 5)^2;$

Exercises:

1. “Add a term to the binomial so that the resulting trinomial becomes a perfect square.

   $a.\ x^2 + 2x;$ $\quad b.\ a^2 + 4ab;$ $\quad c.\ m^2 + 1;$

   $d.\ 9 + 6l;$ $\quad e.\ 10y + 25;$ $\quad f.\ 16x^2 + 8xy;$

2. Complete the full squares:

   $a.\ a^2 + 2x + 2;$ $\quad b.\ x^2 - 2x + 3;$ $\quad c.\ m^2 - 2m - 1;$

   $d.\ 4 + 2q + q^2;$ $\quad e.\ x^2 + 6x + 1;$ $\quad f.\ a^2 - 4a \mp 1;$

   $g.\ m^2 - 6m + 9;$ $\quad h.\ 16 - 8p + p^2;$ $\quad i.\ a^2 - 2a;$

3. Write closed intervals, open intervals, half-open/half closed intervals: