

Classwork 17.



Algebraic identities.

Identities are equalities which are true for any (possible) values of variables.

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

$$(a + b)(a - b) = a^2 - ab + ba - b^2 = a^2 - b^2$$

$$(a + b)^3 =$$

$$(a + b + c)^2 =$$

Factorization of a polynomial is a representation of the polynomial as a product of a monomial and polynomial or a product of two or more polynomials.

For example,

$$6x^2y - 3y^2x = 3xy(2x - y)$$

$$2a^4 + 2a^3x^2 + xa + x^3 = 2a^3(a + x^2) + x(a + x^2) = (a + x^2)(2a^3 + x)$$

Exercises.

1. Simplify the following expressions (combine like terms):

a. $7a + (2a + 3b)$;

b. $9x + (2y - 5x)$;

c. $(5x + 7a) + 4x$;

d. $(5x - 7a) + 5a$;

e. $(3x - 6y) - 4y$;

f. $(2a + 5b) - 7b$;

g. $3m - (5n + 2m)$;

h. $6p - (5p - 3a)$;

- 2.

a. $(x^2 + 4x) + (x^2 - x + 1) - (x^2 - x)$;

b. $(a^5 + 5a^2 + 3a - a) - (a^3 - 3a^2 + a)$;

c. $(x^2 - 3x + 2) - (-2x - 3)$;

d. $(abc + 1) + (-1 - abc)$;

3. Multiply polynomials.

a. $(a + 2)(a + 2)$;

f. $(a + 1)(a + 3)$;

b. $(3 + y)(y + 4)$;

g. $(c + d)(c - 2d)$;

c. $(3 + x)(3 - x)$;

h. $(y - 2)(3 - y)$;

d. $(x - y)(x + y)$;

i. $(x - m)(x - m)$;

e. $(2a + c)(a + ac)$;

j. $(2d + 3l)(2d + 3l)$;

4. Factorize the following polynomials:

a. $x(1 + b) + y(1 + b)$;

f. $(a + b)a - b(a + b)$;

b. $m(2k - 3) + 2(2k - 3)$;

g. $(x + y)3 - a(x + y)$;

c. $2a(1 - b) - 3(1 - b)$;

h. $a(b + 3) - b(3 + b)$;

d. $7x(x - 2y) - 2(2y + x)$;

i. $a(a + b) + (a + b)$;

e. $2x(x - 2y) + 3y(x + 2y)$;

j. $2x(a - 1) - (a - 1)$;

5. Simplify the expressions:

a. $\left(0.3a^{n+1} - \frac{1}{12}a^n - 0.2a^{n-1}\right) \cdot 24a^n - 6a^n \left(\frac{1}{6}a^{n-1} - a^n + 0.3a^{n+1}\right)$

b. $\left(-1\frac{1}{9}b^{n-1} + \frac{1}{3}b^n - 6b^3\right) \cdot 0.9b^{n+1} - 0.8b^n \left(\frac{7}{8}b^n - b^{n+1} - 1\frac{1}{8}b^4\right)$

c. $(x+1)(x^2-x+1) - (x^2-1)x;$

d. $(a^3-b^3)(a^3+b^3) + (a^2+b^2)(a^4-a^2b^2+b^4);$

6. Reduce the following fractions (for valid variables values):

Example:

$$\frac{8z^2 - 24zd}{4zd^2 - 12d^3} = \frac{8z(z - 3d)}{4d^2(z - 3d)} = \frac{2z}{d^2}; \quad d \neq 0, \quad z \neq 3d$$

a. $\frac{2ab - 8a}{3ab - 12a};$

d. $\frac{3p^2 - 8pq}{24pq - 9p^2};$

b. $\frac{5mn - 2np}{25m - 10c};$

e. $\frac{a^2 + 2ab + b^2}{7a + 7b};$

c. $\frac{8z^2 - 24zd}{4zd^2 - 12d^3};$

e. $\frac{7a - 7b}{a^2 - 2ab + b^2};$