Classwork 16.

Algebra.

We already know what is GCD and LCM for several natural numbers and we know how to find them.

Exercise:

Find GCD (GCF) and LCM for numbers

a. 222 and 345.
b. $2^2 \cdot 3^3 \cdot 5$ and $2 \cdot 3^2 \cdot 5^2$

Can we apply the same strategy to find CF and CM for algebraic expressions? (In this case the concept of GCD and LCM cannot be applied.) For example, can CF and CM be found for expressions $2x^2y^5$ and $4x^3y^2$? x and y are variables and can't be represented as a product of factors, but they itself are factors, and the expression can be represented as a product:

$2x^2y^5 = 2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y,$ $\qquad 4x^3y^3 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot$

$A = (Factors, 2x^2y^5) = \{2, x, x, y, y, y, y, y\}, B = (Factors, 4x^3y^3) = \{ 2, 2, x, x, x, y, y\}$

Common devisors are any product of $A \cap B = \{ 2, x, x, y, y\}$.
What about common multiples? Product of all factors of both numbers (or the product of two numbers) will be the multiple, but minimal common multiple will be the product of the

$$A \cup B = \{2,2, x, x, x, y, y, y, y, y\}$$

$$\frac{2x^2y^5}{2 \cdot x^2y^2} = y^3; \qquad \frac{4x^3y^2}{2 \cdot x^2y^2} = 2x;$$

$$\frac{4x^3y^5}{2 \cdot x^2y^5} = 2x; \qquad \frac{4x^3y^5}{4 \cdot x^3y^3} = y^2;$$

Algebraic fractions are expressions of the form $\frac{A}{B}$ $(B \neq 0)$ (where $B \neq 0$), in which both the numerator and the denominator are algebraic expressions (not necessarily polynomials). For example:

$$\frac{3x^2 + y}{y^2 - 5x + 2}; \qquad \frac{\frac{1}{x} - 3}{y + \frac{1}{y}}$$

Properties of the algebraic fractions:

$$\frac{A}{1} = A; \qquad \frac{A}{B} = \frac{A \cdot C}{B \cdot C} \quad (C \neq 0); \qquad -\frac{A}{B} = \frac{-A}{B} = \frac{A}{-B}$$

How to algebraic fractions can be added?
Firstly, let's review the fraction addition:

$$\frac{2}{21} + \frac{5}{24}$$

The common denominator is $3 \cdot 7 \cdot 8 = 168$

$$\frac{2 \cdot 8}{168} + \frac{5 \cdot 7}{168} = \frac{16 + 35}{168} = \frac{51}{168} = \frac{17 \cdot 3}{56 \cdot 3} = \frac{17}{56}$$

Let's add

$$\frac{a}{b} \quad and \quad \frac{n}{b}, \quad b \neq 0$$

These two fractions have the same denominator, b, which cannot be 0.

$$\frac{a}{b} + \frac{n}{b} = \frac{a+n}{b}$$

Another example:

$$\frac{a}{b} + \frac{c}{d}; \quad b, d \neq 0$$

Denominators are two different variables; the only possible common denominator is their product.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$$

Simplification of algebraic fractions:

Fraction can be reduced (simplified) if both, numerator and denominator, represented as a product, and one or more factors are common factors. It's easy, if numerator and denominator are monomials, or expressions represented as a product:

$$\frac{2a^2b}{4a^5b^2} = \frac{1}{2a^3b}, \quad a, b \neq 0$$

Another examples:

$$\frac{48x^3y^4z^3}{56xy^5z^4} = \frac{6x^2}{7yz}; \quad x, y, z \neq 0$$

$$\frac{2(x-1)}{5(x-1)} = \frac{2}{5}; \quad x \neq 1; \qquad \frac{x-y}{y-x} = -\frac{y-x}{y-x} = -1, \quad y \neq x;$$

Exercises:

1. Add fractions:

Example:

$$\frac{2}{x^2a}+\frac{3}{a^2x}=\frac{2a}{a^2x^2}+\frac{3x}{a^2x^2}=\frac{2a+3x}{a^2x^2},\quad a,x\neq 0$$

$a.\ \frac{1}{a}+\frac{1}{b};$ $b.\ \frac{2}{x}-\frac{3}{y};$ $c.\frac{x}{a}+\frac{y}{b};$

$d.\ \frac{5a}{7}-\frac{b}{x};$ $e.\ \frac{1}{2a}-\frac{1}{3};$ $f.\frac{1}{a}-\frac{1}{bc};$

$g.\ \frac{x+1}{x-1}-\frac{2x}{1-x};$ $h.\ \frac{1}{x-y}-\frac{1}{y-x};$ $i.\ \frac{8a+b}{1-a}-\frac{2a-3b}{a-1};$

$j.\ \frac{m}{ab}+\frac{m}{ac};$ $k.\ \frac{2a-3b}{m}+\frac{4a-5b^2}{mb};$

2. Find the permissible (allowed) values of the variable in the fraction.

$\frac{5}{x-1};$ $\frac{3}{a-11};$ $\frac{-7}{b+3};$ $\frac{2a}{(a-5)(a+3)};$ $\frac{3-x}{x^2-4};$ $\frac{1}{(x^2-9)(x^2+25)};$

3. For the letters M and N, select monomials such that the equality holds

$a.\ 2(M-b)=14a-2b$ $b.\ M\cdot(2a+3b)=-6a-9b$

$c.\ N\cdot(2x-M)=12x^2-18xy;$ $d.\ 3a\cdot(N+3M)=15abc-3ac^2$

4. Simplify the expressions:

a. $\left(0.3a^{n+1}-\frac{1}{12}a^n-0.2a^{n-1}\right)\cdot 24a^n-6a^n\left(\frac{1}{6}a^{n-1}-a^n+0.3a^{n+1}\right)$

b. $\left(-1\frac{1}{9}b^{n-1}+\frac{1}{3}b^n-6b^3\right)\cdot 0.9b^{n+1}-0.8b^n\left(\frac{7}{8}b^n-b^{n+1}-1\frac{1}{8}b^4\right)$

$(x+1)(x^2-x+1)-(x^2-1)x;$

$(a^3-b^3)(a^3+b^3)+(a^2+b^2)(a^4-a^2b^2+b^4);$

$(3+m)(m^2-3m+9)-m(m-2)^2;$

$(p^6-q^3)(p^6+q^3)-(p^8-p^4q^2+q^4)(p^4+q^2).$

Test

1. Evaluate:

   $a.\ \dfrac{(2^3\cdot 2^4)^6}{(2\cdot 2^8)^4};$ $\qquad b.\ \dfrac{12^9}{9^4\cdot 2^{16}};$

2. Simplify:

   $a.\ \dfrac{(a^2)^5 b^6}{a^{11}b^5};$ $\qquad b.\ \dfrac{(x^{-3})^5 y^9}{x^{-16}y^8};$ $\qquad c.\ \left(\dfrac{9a^7b^5}{45a^3b}\right)^4$

3. Represent as fraction:

   $a.\ 0.32;$ $\quad b.\ 0.3\overline{2};$ $\quad c.\ 0.\overline{32};$

4. Simplify the following expressions (combine like terms):

   $a.\ (x^2+4x)+(x^2-x+1)-(x^2-x);$

   $b.\ (a^5+5a^2+3a-a)-(a^3-3a^2+a);$

   $c.\ (x^2-3x+2)-(-2x-3);$

   $d.\ (abc+1)+(-1-abc);$

5. Expand (write without parentheses).

   $a.\ (x+y)(x-y);$ $\qquad b.\ (a+b+1)(a-b);$ $\qquad c.\ (2a+3b)^2$

6. During the year, the prices of strudels were increased twice by 50%, and before New Year they started being sold at half price.
   How much does one strudel cost now, if at the beginning of the year it cost 4 dollars?

7. How many grams of jam with 50 % sugar should be added to 100 g of jam with 30% sugar, to get 35% sugar jam.

8. Evaluate (Answer is 0.1).

$$\frac{0.6+2.4\cdot\left(3-0.7\cdot\frac{5}{7}\right)-7:3\frac{1}{2}}{\left(5\frac{1}{4}\cdot 4-\left(5.9-2.7:\frac{9}{11}\right)\right)\cdot 2\frac{1}{2}}$$

9. Draw three arbitrary triangles, draw all three medians in one triangle, all three bisectors in the second tringle, and all three altitudes in the third.