Classwork 11.

Algebra.

1. Find the sum of the like monomials:
   $a.\ 3m + 5m;$ $b.\ 3b + b + b;$ $c.\ 4ab + ab + 12ab$
   $d.\ 15a^2b + 14a^2b + 7a^2b;$ $e.\ 25b^2c^2 + (-27)b^2c^2;$
   $f.\ a^2bc + 2abca + (-3bca^2);$ $g.\ (-aba^2) + 7a^2ba + a^3b$
   $h.\ 7a^2 + (-3a^2) + (-4a^2)$

What can you say about the result, is it monomoal or polynomial?

Properties of polynomials:

Property 1. The terms of a polynomial can be rearranged.
In other words, if one polynomial differs from another only in the order of its terms, then these two polynomials are considered equal. For example:

$$2a^2b + 3ab^2 = 3ab^2 + 2a^2b$$

$$a^2 - b^2 = -b^2 + a^2$$

$$x^2 - x + 1 = 1 - x + x^2$$

Property 2. Adding zero (the zero polynomial) to a polynomial does not change it.
In other words, if one polynomial is obtained from another by adding the number “zero,” then these two polynomials are considered equal. For example:

$$a^4 + (-a^2) + 0 = a^4 + (-a^2)$$

$$0 + abc = abc$$

$$2a - 3b + 0 - c = 2a - 3b - c$$

Property 3. In a polynomial, like terms can be combined.
In other words, if one polynomial is obtained from another by replacing like terms with their sum, then these two polynomials are considered equal. For example:

$$a^2 + ab - ab + b^2 = a^2 + 1 \cdot ab + (-1) \cdot ab + b^2 = a^2 + \big(1 + (-1)\big) \cdot ab + b^2$$
$$= a^2 + 0 \cdot ab + b^2 = a^2 + b^2$$

A polynomial is said to be in standard form if all its terms are written in standard form and there are no like terms among them.

Let's give examples of polynomials in standard form:

$$2, \quad a, \quad a - b, \quad a^2 + 2ab^2 + b^2, \quad \frac{1}{7} - a, \quad 0.$$

The following polynomials can serve as examples of non-standard form polynomials:

$$a \cdot a - 5a + 6, \quad a^3 - 2ab + b^2 - 3ab - 11, \quad 3 - 5 + a^2$$

In the first of these, not all terms are written in standard form. The second and third have like terms.

A polynomial in standard form consisting of two terms is called a binomial; a polynomial in standard form consisting of three terms is called a trinomial, and so on.

Any polynomial can be reduced to standard form. For this, it is necessary to:

- reduce each of its terms to standard form;
- combine like terms.

Example:

$$a^3 + 2aba + b^2a + ba^2 - 2abb - b^2b = a^3 + 2a^2b + ab^2 + a^2b - 2ab^2 - b^3$$
$$= a^3 + 3a^2b - ab^2 - b^3$$

Exercises:

Geometry.

Triangles.

Triangles can be classified by two parameters:

- Side length: **equilateral** (all sides equal), **isosceles** (at least two sides equal), or **scalene** (no sides equal).
- Angle measure: **acute** (all angles less than right angle), **right** (one angle is right angle), or **obtuse** (one angle is greater than the right angle).

A **median** of a triangle is a segment drawn from a **vertex** to the **midpoint of the opposite side**.

An **angle bisector** is a ray (segment in a triangle) that **divides an angle into two equal angles**.

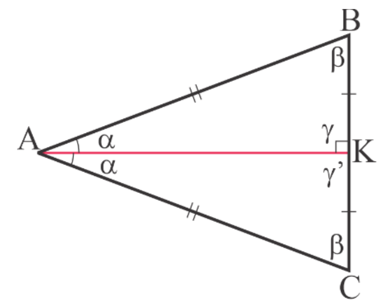
An **altitude** of a triangle is a segment drawn from a **vertex perpendicular** to the opposite side (or its extension).

The greater angle of a triangle lies opposite the greater side.

Let the side BC of triangle ABC be greater than side AC. We lay off a segment CA’ equal to AC on side BC. Since CA’ = AC < BC, the point A’ lies on the segment BC. Therefore, the ray AA’ passes between the sides of angle A. Consequently, angle ∠A> ∠CAA’ .

Theorem:

In an isosceles triangle, a bisector, drawn to the base, is a median and an altitude.



Given: Triangle ABC is isosceles,

$$|AB| = |AC|$$

Segment [AK] is the angle bisector, so $\angle BAK = \angle KAC$,

To prove:

1. [AK] is a median ($|CK| = |KB|$)
2. [AK] is an altitude ($\angle BKA = \angle AKC = 90°$).

Proof:

Consider triangles Δ ABK and ΔACK are congruent by the SAS test: $|AB| = |AC|$, because triangle ABC is isosceles, segment [AK] is a common side, $\angle BAK = \angle KAC$, by construction.

Therefore, $|CK| = |KB|$, and hence [AK] is a median. Also, $\angle ABK = \angle ACK$. Angles at the base of the isosceles triangle are equal.

$$\alpha + \beta + \gamma = \alpha + \beta + \gamma' = 180^{\circ}$$

Thus,

$$2\alpha + 2\beta = 180 \Rightarrow \alpha + \beta = 90°, \quad \Rightarrow \text{means "therefore"}$$

And

$$90 + \gamma = 90 + \gamma' = 180° \Rightarrow \gamma = \gamma' = 90°$$

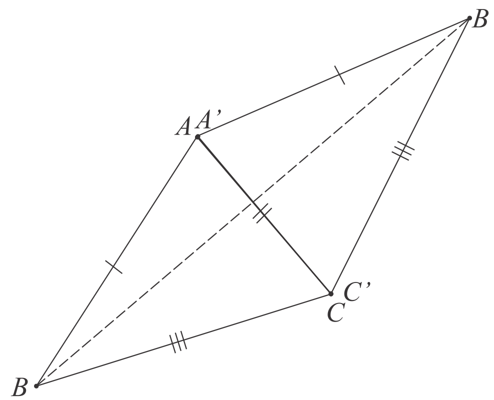
[AK] is an altitude.

SSS (Side-Side-Side): If three pairs of sides of two triangles are equal in length, then the triangles are congruent.

Let ΔABC and ΔA'B'C' be two triangles such that

$AC = A'C', AC = A'C', BC = B'C'$.

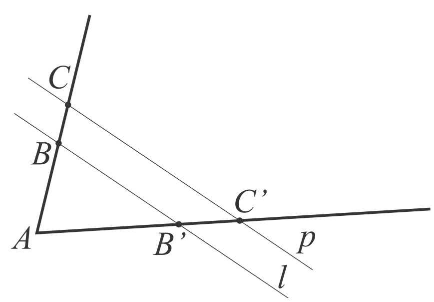
It is required to prove that triangles are congruent. Proving this test by superimposing, the same way as we proved the first two tests, turns out to be awkward, because knowing nothing about the measure of angles, we would not be able to conclude from coincidence of two corresponding sides that the other sides coincide as well. Instead of superimposing, let us apply *juxtaposing*.



Juxtapose ΔABC and $\Delta A'B'C'$ in such a way that their congruent sides AC and $A'C'$ would coincide and the vertices B and B' would lie on the opposite sides of $A'C'$ (see the picture). Connecting vertices B and B' we will get 2 isosceles triangles, BAB' and BCB'. In the isosceles triangle angles at the base are congruent, so $\angle ABB' = \angle AB'B$, and $\angle CBB' = \angle CB'B$, therefore $\angle ABC = \angle AB'C$ and triangle ΔABC is congruent to the triangle $\Delta A'B'C'$.

Congruency tests.

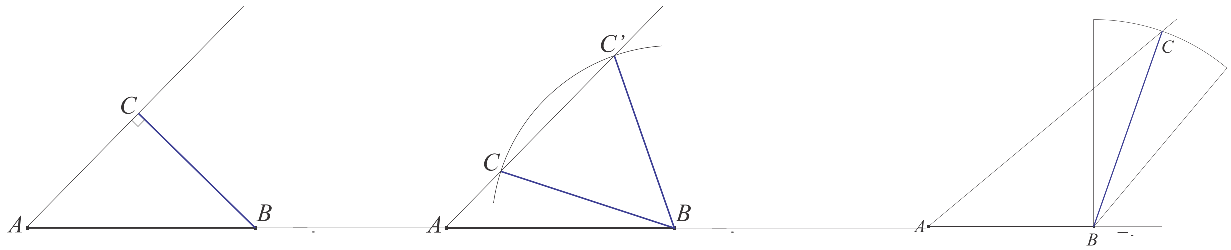
- **SAS** (Side-Angle-Side): If two pairs of sides of two triangles are equal in length, and the included angles are equal in measurement, then the triangles are congruent.
- **SSS** (Side-Side-Side): If three pairs of sides of two triangles are equal in length, then the triangles are congruent.
- **ASA** (Angle-Side-Angle): If two pairs of angles of two triangles are equal in measurement, and the included sides are equal in length, then the triangles are congruent.



Based in these criteria, we can see that a triangle is defined by either three sides, or by the side and two adjacent angles, or by the two sides and the angle formed by them. And what about another combination of sides and angles? Do three angles define a triangle? Are the two triangles with congruent angles are congruent? No, just see the example on the picture. Two parallel lines l and p intersect two sides of the angle $\angle A$. Two triangles $\Delta ABB'$ and $\Delta ACC'$ are formed. Angle A is the common angle, angles $\angle ABB'$ and $\angle ACC'$ are congruent, as well as angles $\angle AB'B$ and $\angle AC'C$ as the corresponding angles formed by transversal crossing two parallel lines. Triangles $\Delta ABB'$ and $\Delta ACC'$ are not congruent.

Let's take a look on the **AAS** combination, two angles and the side, not adjacent to both angles. This case can easy be reduced to **ASA** criteria, since the third angle is always known.

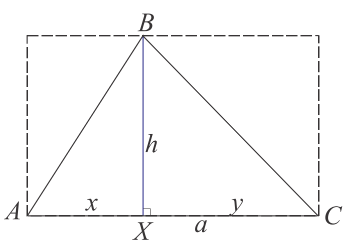
SSA (two sides and the angle not formed by these two sides) condition is more interesting, since several cases can be considered.



The first case represents the shortest possible second side and the right triangle is formed, second case represents the situation where the second side is bigger than the distance from point B to the ray AC', but smaller than the length of the segment AB. Two triangles are satisfying the condition SSA. The third case shows that if the length of the second side is equal or bigger than the length of the segment AB, the only one triangle satisfy the SSA condition.

Area of a triangle.

The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude. For the acute triangle it is easy to see.

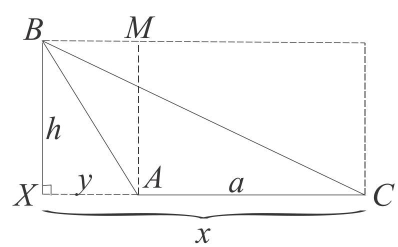


$$S_{rec} = h \times a = x \times h + y \times h \qquad S_{\Delta} = \frac{1}{2} h \times a$$

$$S_{\Delta ABX} = \frac{1}{2} h \times x, \qquad S_{\Delta XBC} = \frac{1}{2} h \times y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

$$S_{\Delta ABC} = \frac{1}{2} h \times x + \frac{1}{2} h \times y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$

For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex.



$$S_{\Delta XBC} = \frac{1}{2} h \times x, \qquad S_{\Delta XBA} = \frac{1}{2} h \times y$$

Exercises:

1. What properties of polynomials were used in simplifying the polynomial:

   a. $a + b - a = a - a + b = 0 + b = ba$

   b. $2x - y + x - 3y - 5x = 2x + x - 5x - y - 3y = (2 + 1 - 5)x - (1 + 3)y$
   $= -2x - 4y$

2. Reduce the polynomial to its standard form, and determine its degree:

   $a.\ \ 4a^2b + 5b^2b + baa + 3aba$

   $b.\ \ 5a^3 - 7ax^3 - 2ax^3 - a^3x - ax^3$

   $c.\ \ 3ax^2 - 3a^2x + 2a^2x^2 - 7a^2x^2 - a^2x$

   $d.\ \ 6n^3 - 8p^2n^3 + p^2n^2 + 12n^3p^2 + 2\,n^3$

3. For the letters M and N, select monomials such that the equality holds

$(a + b + c) + (M - N + c) = 4a - 2b + 2c;$
$(7x - N) - (M + 2y) = 3x - 2y;$
$(M + N) - (2a - b) + (a - 4b) = 5a + 7b;$
$(a - M) - (N + 7b) - (2a + b) = -5a - 10b.$

4. Simplify the expressions:

   a. $\left(0.3a^{n+1} - \frac{1}{12}a^n - 0.2a^{n-1}\right) \cdot 24a^n - 6a^n\left(\frac{1}{6}a^{n-1} - a^n + 0.3a^{n+1}\right)$

   b. $\left(-1\frac{1}{9}b^{n-1} + \frac{1}{3}b^n - 6b^3\right) \cdot 0.9b^{n+1} - 0.8b^n\left(\frac{7}{8}b^n - b^{n+1} - 1\frac{1}{8}b^4\right)$

5. Instead of * find monomial such that the equality became an identity.

   a. $* \cdot (a^2 + 2ab) = 1.7a^3 + 3.4a^3b$

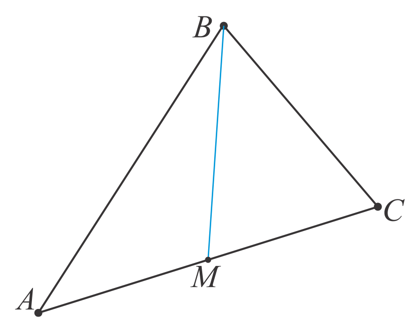
   b. $(0.3ax - 0.1a^2x + a) \cdot * = 0.6a^2x^2 - 0.2a^3x^2 + \dots$

6. What is a degree, highest coefficient ant constant term of the polynomial identically equal to

   $a.\ \ (-2x^3 - 3x^2 + x - 1)(3x^2 - x - 2)$

   $b.\ \ (x^5 - 5)(-2x^6 - x^3 - 2)$

   $c.\ \ (x^n - 3x^{n-1} + \cdots + 2x + 3)(-x^n + x^{n-1} - x^{n-2} + \cdots + x + 1)$



7. Segment BM in the triangle ABC on the picture below, is a median. Prove, that the area of the triangle AMB is equal to the area of the triangle MBC.

8. Through the midpoint O of segment [AB], a line is drawn perpendicular to line AB. Prove that every point on this line is equally distant from points A and B.

9. Prove the congruence of the triangles by the median and the angles into which the median divides the angle of the triangle.