Classwork 10.

Algebra.

Monomials.

Monomial is a product of variables in nonnegative integer power and a number, which is called a coefficient. For example: $xy^3 6,\ 56,\ 3c^5\ d^{10}, 2x3y5$.
Usually, monomials are written in the following form: first goes a coefficient (only one number), then the variable with the highest power and so on…Example above $6y^3x, 56, 3d^{10}c^5, 30xy.$
Degree of a monomial is the sum of all exponents of variables. The degree of $6y^3x$ is 4 $(1+3=4)$.

Polynomials.

The sum of monomials is a polynomial.

The degree of a polynomial is the highest of the degrees of its monomials (individual terms) with non-zero coefficients. The degree of a term is the sum of the exponents of the variables that appear in it, and thus is a non-negative integer. The term order has been used as a synonym of degree but, nowadays, may refer to several other concepts (see order of a polynomial (disambiguation)). For example, the polynomial $7x^2y^3+4x-9$ which can also be expressed as $7x^2y^3+4x^1y^0-9x^0y^0$ has three terms. The first term has a degree of 5 (the sum of the powers 2 and 3), the second term has a degree of 1, and the last term has a degree of 0. Therefore, the polynomial has a degree of 5, which is the highest degree of any term.

To determine the degree of a polynomial that is not in standard form (for example: $(x+1)^2-(x-1)^2$, one has to put it first in standard form by expanding the products (by distributivity) and combining the like terms; $(x+1)^2-(x-1)^2=4x$ is of degree 1, even though each summand has degree 2.

Identities are true equalities for any possible values of variables.

$$(a+b)^2=(a+b)(a+b)=a^2+ab+ab+b^2=a^2+2ab+b^2$$
$$a^2-b^2=(a-b)(a+b)=a^2+ab-ab-b^2$$

How we can get these identities by factorization of poynomials?

$a^2+2ab+b^2=a^2+ab+ab+b^2=a(a+b)+b(a+b)=(a+b)(a+b)=(a+b)^2$
$a^2-b^2=a^2+ab-ab-b^2=a(a+b)-b(a+b)=(a-b)(a+b)$

Geometry.

Triangles.

Triangles can be classified by two parameters:

- Side length: **equilateral** (all sides equal), **isosceles** (at least two sides equal), or **scalene** (no sides equal).

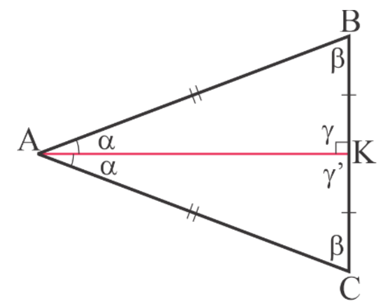
- Angle measure: **acute** (all angles less than right angle), **right** (one angle is right angle), or **obtuse** (one angle is greater than the right angle).

A **median** of a triangle is a segment drawn from a **vertex** to the **midpoint of the opposite side**.

An **angle bisector** is a ray (segment in a triangle) that **divides an angle into two equal angles**.

An **altitude** of a triangle is a segment drawn from a **vertex perpendicular** to the opposite side (or its extension).

Theorem:



In an isosceles triangle, a bisector, drawn to the base, is a median and an altitude.

Given: Triangle ABC is isosceles,

$$|AB| = |AC|$$

Segment [AK] is the angle bisector, so $\angle BAK = \angle KAC$,

To prove:

1. [AK] is a median ($|CK| = |KB|$)
2. [AK] is an altitude ($\angle BKA = \angle AKC = 90°$).

Proof:

Consider triangles Δ ABK and ΔACK are congruent by the SAS test: $|AB| = |AC|$, because triangle ABC is isosceles, segment [AK] is a common side, $\angle BAK = \angle KAC$, by construction.

Therefore, $|CK| = |KB|$, and hence [AK] is a median. Also, $\angle ABK = \angle ACK$. Angles at the base of the isosceles triangle are equal.

$$\alpha + \beta + \gamma = \alpha + \beta + \gamma' = 180^{\circ}$$

Thus,

$$2\alpha + 2\beta = 180 \Rightarrow \alpha + \beta = 90°, \quad \Rightarrow \text{means "therefore"}$$

And

$$90 + \gamma = 90 + \gamma' = 180° \Rightarrow \gamma = \gamma' = 90°$$

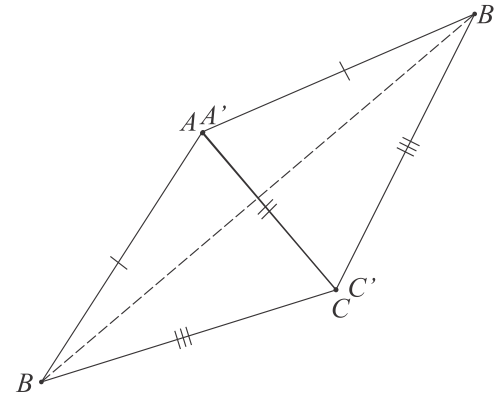
[AK] is an altitude.

SSS (Side-Side-Side): If three pairs of sides of two triangles are equal in length, then the triangles are congruent.

Let ΔABC and ΔA'B'C' be two triangles such that

$AC = A'C', AC = A'C', BC = B'C'$.

It is required to prove that triangles are congruent. Proving this test by superimposing, the same way as we proved the first two tests, turns out to be awkward, because knowing nothing about the measure of angles, we would not be able to conclude from coincidence of two corresponding sides that the other sides coincide as well. Instead of superimposing, let us apply *juxtaposing*.

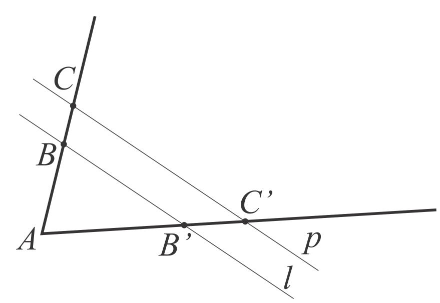


Juxtapose ΔABC and $\Delta A'B'C'$ in such a way that their congruent sides AC and $A'C'$ would coincide and the vertices B and B' would lie on the opposite sides of $A'C'$ (see the picture). Connecting vertices B and B' we will get 2 isosceles triangles, BAB' and BCB'. In the isosceles triangle angles at the base are congruent, so $\angle ABB' = \angle AB'B$, and $\angle CBB' = \angle CB'B$, therefore $\angle ABC = \angle AB'C$ and triangle ΔABC is congruent to the triangle $\Delta A'B'C'$.

Congruency tests.

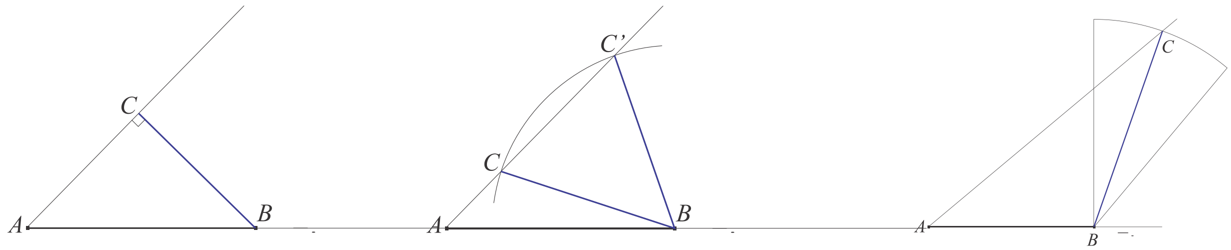
- **SAS** (Side-Angle-Side): If two pairs of sides of two triangles are equal in length, and the included angles are equal in measurement, then the triangles are congruent.
- **SSS** (Side-Side-Side): If three pairs of sides of two triangles are equal in length, then the triangles are congruent.
- **ASA** (Angle-Side-Angle): If two pairs of angles of two triangles are equal in measurement, and the included sides are equal in length, then the triangles are congruent.

Based in these criteria, we can see that a triangle is defined by either three sides, or by the side and two adjacent angles, or by the two sides and the angle formed by them. And what about another combination of sides and angles? Do three angles define a triangle? Are the two triangles with congruent angles are congruent? No, just see the example on the picture. Two parallel lines l and p intersect two sides of the angle $\angle A$. Two triangles $\Delta ABB'$and $\Delta ACC'$ are formed. Angle A is the common angle, angles $\angle ABB'$and $\angle ACC'$ are congruent, as well as angles $\angle AB'B$ and $\angle AC'C$ as the corresponding angles formed by transversal crossing two parallel lines. Triangles $\Delta ABB'$and $\Delta ACC'$ are not congruent.



Let's take a look on the **AAS** combination, two angles and the side, not adjacent to both angles. This case can easy be reduced to **ASA** criteria, since the third angle is always known.

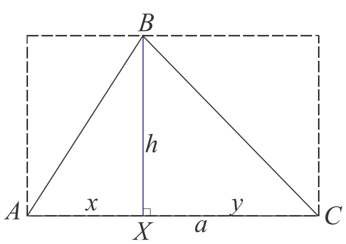
SSA (two sides and the angle not formed by these two sides) condition is more interesting, since several cases can be considered.



The first case represents the shortest possible second side and the right triangle is formed, second case represents the situation where the second side is bigger than the distance from point B to the ray AC', but smaller than the length of the segment AB. Two triangles are satisfying the condition SSA. The third case shows that if the length of the second side is equal or bigger than the length of the segment AB, the only one triangle satisfy the SSA condition.

Area of a triangle.

The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude. For the acute triangle it is easy to see.

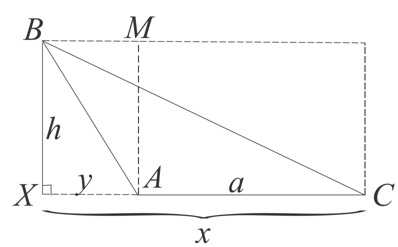


$$S_{rec} = h \times a = x \times h + y \times h \qquad S_{\Delta} = \frac{1}{2} h \times a$$

$$S_{\Delta ABX} = \frac{1}{2} h \times x, \qquad S_{\Delta XBC} = \frac{1}{2} h \times y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

$$S_{\Delta ABC} = \frac{1}{2} h \times x + \frac{1}{2} h \times y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$

For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex.



$$S_{\Delta XBC} = \frac{1}{2} h \times x, \qquad S_{\Delta XBA} = \frac{1}{2} h \times y$$

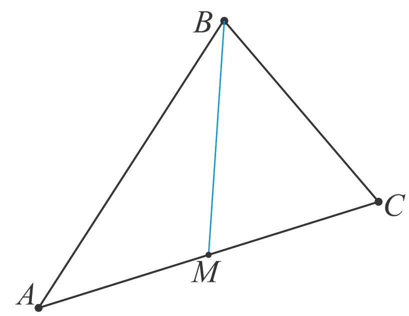
Exercises:

1. Simplify the expressions:
   a. $\left(0.3a^{n+1} - \frac{1}{12} a^n - 0.2a^{n-1}\right) \cdot 24a^n - 6a^n \left(\frac{1}{6} a^{n-1} - a^n + 0.3a^{n+1}\right)$

b. $\left(-1\frac{1}{9}b^{n-1}+\frac{1}{3}b^{n}-6b^{3}\right)\cdot 0.9b^{n+1}-0.8b^{n}\left(\frac{7}{8}b^{n}-b^{n+1}-1\frac{1}{8}b^{4}\right)$

2. Instead of * find monomial such that the equality became an identity.
   a. $*\cdot(a^2+2ab)=1.7a^3+3.4a^3b$
   b. $(0.3ax-0.1a^2x+a)\cdot * = 0.6a^2x^2-0.2a^3x^2+\ldots$

3. What is a degree, highest coefficient ant constant term of the polynomial identically equal to
   a. $(-2x^3-3x^2+x-1)(3x^2-x-2)$
   b. $(x^5-5)(-2x^6-x^3-2)$
   c. $(x^n-3x^{n-1}+\cdots+2x+3)(-x^n+x^{n-1}-x^{n-2}+\cdots+x+1)$



4. Segment BM in the triangle ABC on the picture below, is a median. Prove, that the area of the triangle AMB is equal to the area of the triangle MBC.

5. Through the midpoint O of segment [AB], a line is drawn perpendicular to line AB. Prove that every point on this line is equally distant from points A and B.

6. Prove the congruence of the triangles by the median and the angles into which the median divides the angle of the triangle.