MATH 6 [2026 MAY 3]
HANDOUT 26 : FACTORIZATION AND POLYNOMIAL DIVISION

FACTORIZATION

When handling with large algebraic expressions, it is often possible to simplify them. One of of doing this is by **factorization**. As its name suggests, this method consists of finding a common factor in two or more terms. For example, in the following expression

$$7x + 9x - 5x$$

the three terms share the common factor x. Therefore, we can rewrite this expression as:

$$7x + 9x - 5x = (7 + 9 - 5)x = 11x.$$

In general, we will have the following identities:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

$$ab + ac = a(b + c)$$

POLYNOMIAL DIVISION

In arithmetics, we divide numbers using long division. We can do the similar process for dividing algebraic expressions. For example, we can divide $(2x^2 - 7x + 3)$ by $(x - 3)$

$$\begin{array}{r|r}
 & 2x - 1 \\ \hline
x - 3 & 2x^2 - 7x + 3 \\
 & -2x^2 + 6x \\ \hline
 & -x + 3 \\
 & x - 3 \\ \hline
 & 0
\end{array}$$

This means that

$$2x^2 - 7x + 3 = (x - 3)(2x - 1)$$

Now, if we want to solve an equation

$$2x^2 - 7x + 3 = 0$$

we can equate both factors to 0, and get two solutions: when $x - 3 = 0$, then $x = 3$, and when $2x - 1 = 0$, then $x = 1/2$.

This can be useful if we want to solve an equation for which we don't have any good method. For example, if we are trying to solve an equation

$$2x^2 + 7x - 22 = 0$$

we can try to guess one of the solution using guess-and-check method. When we check small whole numbers, we can see that $x = 2$ is a solution, and that would mean that we can divide $2x^2 + 7x - 22$ by $x - 2$:

$$\begin{array}{r|r}
 & 2x + 11 \\ \hline
x - 2 & 2x^2 + 7x - 22 \\
 & -2x^2 + 4x \\ \hline
 & 11x - 22 \\
 & -11x + 22 \\ \hline
 & 0
\end{array}$$

This means that

$$2x^2 + 7x - 22 = (x-2)(2x+11)$$

and to solve this equation we have to equate two factors to 0: $x - 2 = 0$ gives us $x = 2$, and $2x + 11 = 0$ gives us $x = -5.5$. Note that as with division of numbers, in some cases you may end up with a remainder:

$$\begin{array}{r|rrr}
 & & & 2x & +15 \\
\hline
x-3 & 2x^2 & +9x & -22 \\
 & -2x^2 & +6x & \\
\cline{2-4}
 & & 15x & -22 \\
 & & -15x & +45 \\
\cline{3-4}
 & & & 23
\end{array}$$

CLASSWORK

1. Solve the following inequalities:

(a) $\frac{x}{x-2} < 3$
(b) $(x+4)(3-x) > 0$
(c) $(2-x)(1-2x) > 0$
(d) $\frac{x}{x+3} > 2$

2. Factor and solve equations

(a) $3x^2 - 5x + 2 = 0$
(b) $x^2 - 5x + 6 = 0$
(c) $5x^2 - 6x + 1 = 0$
(d) $x^3 + 4x^2 - 5x = 0$

3. Perform long division

(a) $(2x^3 - x^2 - 5x - 2)$ by $(x-2)$
(b) $(3x^3 + x^2 - 12x - 4)$ by $(3x+1)$
(c) $(2x^4 - x^3 + 4x^2 - 1)$ by $(2x-1)$
(d) $(x^3 + 2x^2 - 4x - 3)$ by $(x+3)$

HOMEWORK

1. Factor:

(a) $6a + 12 =$
(b) $mn + n =$
(c) $5xy - 15x =$
(d) $4ax - 8ax^2 + 12ax^3 =$

2. Factor using the factorization identities we learned above:

(a) $9 - x^2 =$
(b) $x^6 - 4 =$
(c) $9 - 6x + x^2 =$
(d) $a^3 - 2a^2x + ax^2 =$

3. Show that the left hand side (LHS) = right hand side (RHS):

(a) $x^2(x+1) - x - 1 = x(x+1)^2 - (x+1)^2$
(b) $2x(x+b) + a(x+b) = (2x+a)x + (2x+a)b$

4. Complete the long division of polynomials:

(a) $(x^2 - 3x - 4)$ by $(x-4)$
(b) $(x^3 - 2x^2 + 2x - 4)$ by $(x-2)$
(c) $(x^4 + 3x^3 - x^2 - x + 6)$ by $(x+3)$
(d) $(2x^4 - 5x^3 + 2x^2 + 5x - 10)$ by $(x-2)$

5. Solve the following equations by first guessing one of the solutions, and then using polynomial division.

(a) $2x^2 + 3x - 14 = 0$
(b) $3x^2 - 10x + 3 = 0$
(c) $5x^2 + 8x - 4 = 0$
(d) $3x^2 - x - 4$

6. Solve the following inequalities:

(a) $\frac{x}{x+1} > 2$
(b) $(1-x)(2x+1) < 0$
(c) $(3-4x)(x+2) > 0$
(d) $\frac{x-1}{2x+3} < 1$