MATH 6 [2026 APR 19]
HANDOUT 24 : VECTORS II; COORDINATES

VECTORS IN COORDINATES

Vectors (directed segments) are very simple to work with in coordinates. Any vector can be written as a pair of numbers: its parts along axis x and along axis y:

$$\vec{v} = \overrightarrow{AB} = (v_x, v_y)$$

these numbers are called "coordinate components", or simply components, of a vector, and typically written using subscripts x, y. The zero vector has components $\overrightarrow{0} = (0, 0)$.

There is a simple rule to find coordinates of a vector from point $A(x_A, y_A)$ to point $B(x_B, y_B)$:

$$\overrightarrow{AB} = (x_B - x_A, y_B - y_A)\,.$$

Note that this is similar to the triangle rule for subtracting the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$,

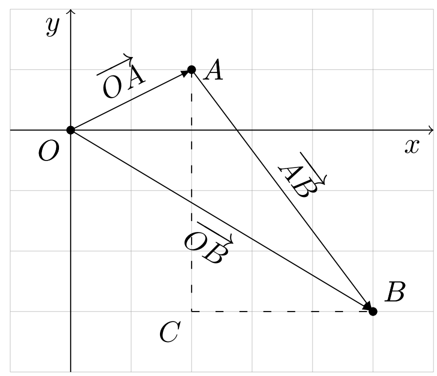
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}\,.$$

For example, a vector from $A(2,1)$ to $B(5,-3)$ will have components

$$\overrightarrow{AB} = (5-2, -3-1) = (3, -4)\,.$$

To compute the length of a vector, use the Pythagoreal theorem for triangle ABC:

$$|\overrightarrow{AB}| = \sqrt{3^2 + (-4)^2} = 5\,.$$



OPERATIONS WITH VECTORS IN COORDINATES

To add or subtract vectors $\vec{a} = (a_x, a_y)$ and $\vec{b} = (b_x, b_y)$, add or subtract their components:

$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y),$$
$$\vec{a} - \vec{b} = (a_x - b_x, a_y - b_y)$$

To multiply a vector by a number, multiply its components by a number

$$m\vec{a} = m(a_x, b_x) = (ma_x, mb_x)\,.$$

The resulting vector $m\vec{a}$ is parallel to $\vec{a}$. *The reverse is also true: if two vectors $\vec{a}$ and $\vec{d}$ are paralell, then there must be some number m such that*

$$\vec{d} = m\vec{a}\,,$$

Of course, vector $\vec{a}$ has to be a nonzero vector (otherwise all $m\vec{a}$ are zero).

VECTORS AND LINES

If a line is parallel to a nonzero vector $a = (a_x, a_y)$, and two points $C(x_C, y_C)$, $D(x_D, y_D)$ are on that line, then vector $\overrightarrow{CD}$ must be parallel to $\vec{a}$. There is a rule that if two vectors are parallel, then there must be some number m

$$\overrightarrow{CD} = m\,\vec{d} \quad \Leftrightarrow \quad \begin{cases} x_D - x_C &= ma_x\,, \\ y_D - y_C &= ma_y\,, \end{cases}$$

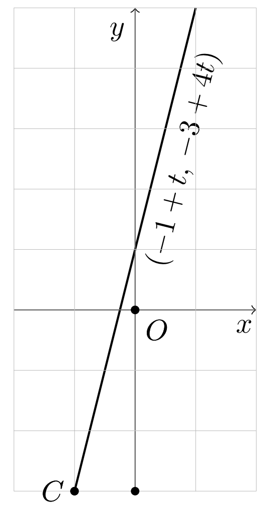
And so is for any point $P(x, y)$ on the same line, there is a number t,

$$\begin{cases} x - x_C &= ta_x\,, \\ y - y_C &= ta_y\,, \end{cases}$$

This number t is called a "parameter" of the line equation, and changing $-\infty < t < \infty$ we can trace out the entire line, even if $a_x = 0$ and the line is vertical. If $a_x \neq 0$, we can eliminate t,

$$y - y_C = \frac{a_y}{a_x}(x - x_C)$$

and get the usual equation of a line with slope a_y/a_x.



Example Line parallel to vector $\vec{a} = (a_x, a_y) = (4, 1)$ will have slope $a_y/a_x = 4$. Such line throught point $C(-1, -3)$ will have equation

$$y - y_C = (\text{slope})(x - x_C) \quad \Leftrightarrow \quad y - (-3) = 4\big(x - (-1)\big) \quad \Leftrightarrow \quad y = 4x + 1$$

Alternatively, any point $P(x, y)$ on the line must have coordinates

$$(x, y) = (x_C, y_C) + t(a_x, a_y) = (-3 + 4t, -1 + t)$$

and for any number t the point (x, y) is on the line.

Homework

1. Let $A(3, 6)$, $B(5, 2)$, and $O(0, 0)$ is the origin
  (a) Find coordinates of vector $\vec{v} = \overrightarrow{AB}$.
  (b) Find coordinates of point P if $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{2}\vec{v}$
  (c) Find coordinates of point Q if $\overrightarrow{OQ} = \overrightarrow{OA} + 2\vec{v}$

2. A grasshopper sitting at point $A(2, 3)$ makes 4 jumps: first $(5, 1)$, then $(-3, 2)$, third $(1, -4)$, and after the fourth it lands at the origin $(0, 0)$.
  (a) What was the fourth jump?
  (b) How far from the origin was the grasshopper after the second jump?
  (c) What was the total distance the grasshoper jumped?

3. Consider a parallelogram $ABCD$ with vertices $A(1, 2)$, $B(4, 8)$, and $D(6, 0)$. Find the coordinates of
  (a) vertex C ;
  (b) the midpoint of segment BD ;
  (c) the midpoint of segment AC .

4. A quadrilateral $ABCD$ has vertices $A(-3, 5)$, $B(6, 1)$, $C(4, -3)$, and $D(-4, -4)$.
  - Find the coordinates of the midpoints of segments $E \in AB$, $F \in BC$, $G \in CD$, and $H \in AD$.
  - Show that $EFGH$ is a parallelogram.
  - Find the point at which its diagonals of $EFGH$ intersect.

5. Consider a line that has equation $y = \frac{3}{4}x - \frac{1}{2}$.
  (a) Show that vector $\vec{v} = (-4, -3)$ is parallel to the line.
  (b) Show that vector $\vec{w} = (6, -8)$ is perpendicular to the line.