MATH 6 [2026 MAR 29]
HANDOUT 22 : CARTESIAN COORDINATES IV:
TRANSFORMATIONS; LINEAR INEQUALITIES

Transformations of figures and graphs in coordinates

Any figure that is "drawn" using equation

$$y = \{\text{some formula in terms of }\}(x)$$

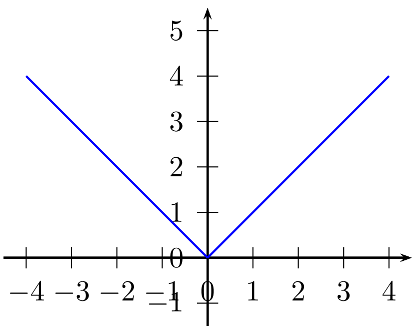
can be transformed by making some changes to x and y variables in that formula. For example, let's take a "wedge" $y = |x|$:

- **to shift the graph horizontally to the right** by $a > 0$, replace $x \to (x - a)$:

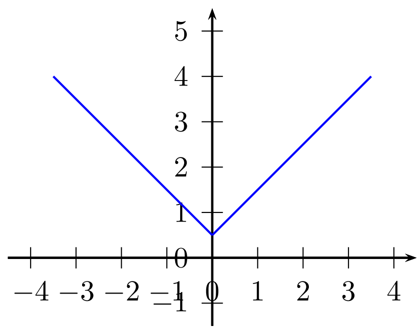
  $$y = |x| \quad \to \quad y = |x - a|$$

  naturally, to shift to the left, just make a negative shift by $(-a) < 0$, to get $y = |x + a|$;
- **to shift the graph vertically up** by $b > 0$, add b to the right hand side,

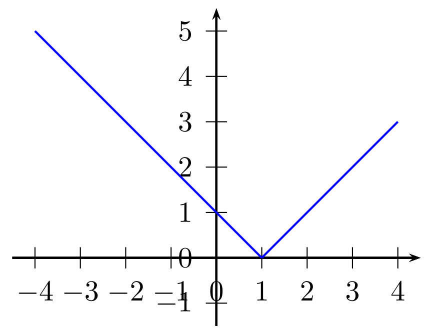
  $$y = |x| \quad \to \quad y = |x| + b \quad \text{(or, which is the same, replace } y \to (y-b) : y - b = |x|\,.)$$



(wedge y=$|x|$)



(shifted by 0.5 up)

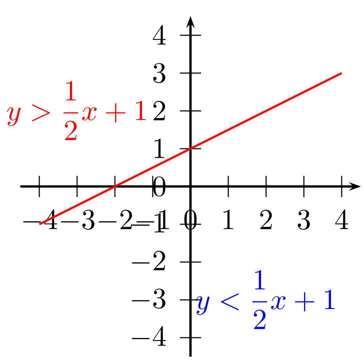


(shifted by 1 right)

One can also

- **to stretch the graph horizontally** by k, replace $x \to x/k$;
  if $k < 0$, then the graph is also flipped left-right;
- **to stretch the graph vertically** by k, replace $y \to y/k$;
  if $k < 0$, then the graph is also flipped up-down;
- **to rotate the graph by** 90^o **about the origin counter-clockwise (CCW)**, replace $x \to y$, $y \to (-x)$;

Inequalities on coordinate plane



A line divides (coordinate) plane in to two half-planes.
For example, line

$$y = \frac{1}{2}x + 1$$

divides the entire plane into two halfplanes,

$$\text{above the line} : \left\{\forall(x,y) : y > \frac{1}{2}x + 1\right\}$$

$$\text{below the line} : \left\{\forall(x,y) : y < \frac{1}{2}x + 1\right\}$$

Example Find inequalities that all points inside quadrilateral $ABCD$ satisfy, if $A(0,2)$, $B(3,3)$, $C(5,1)$, and $D(3,-1)$. **Solution** From a figure, all the points should lie below lines (AB) and (BC) but above lines (CD) and (AD):

$$\begin{aligned}
\text{below } (AB): &\quad y-2 < \frac{3-2}{3-0}\cdot x \quad \Leftrightarrow \quad y < \frac{1}{3}x+2\,,\\
\text{below } (BC): &\quad y-3 < \frac{1-3}{5-3}\cdot(x-3) \quad \Leftrightarrow \quad y < -x+6\,,\\
\text{above } (CD): &\quad y+1 > \frac{1+1}{5-3}\cdot(x-3) \quad \Leftrightarrow \quad y > x-4\,,\\
\text{above } (AD): &\quad y+1 > \frac{-1-2}{3-0}\cdot(x-3) \quad \Leftrightarrow \quad y > -x+2\,.
\end{aligned}$$

CLASSWORK

1. Check that the graph of the equation $|x|+|y|=1$ is the square $ABCD$ with corners at $A(0,1)$, $B(1,0)$, $C(0,-1)$, and $D(-1,0)$.
2. What is the equation for a rhombus with corners at $(0,2)$, $(1,0)$, $(0,-2)$, and $(-1,0)$.
3. What is the equation for a rhombus with corners at $(1,3)$, $(2,1)$, $(1,-1)$, and $(0,1)$?

HOMEWORK

1. Check that the graph of the equation
$$|x+y|+|x-y|=2$$
is the square $ABCD$ with vertices at $A(1,1)$, $B(1,-1)$, $C(-1,-1)$, and $D(-1,1)$.
Hint: consider the 4 regions of the plane where $y>x$ or $y<x$ and $y>(-x)$ or $y<(-x)$.
2. What is the graph of the rectangle with vertices at
(a) $(3,2)$, $(3,-2)$, $(-3,-2)$, and $D(-3,2)$?
(b) $(0,1)$, $(3,1)$, $(3,0)$, and $D(0,0)$?
Hint: transform the equation from the previous problem.
3. Draw a quadrilateral with vertices $A(1,1)$, $B(4,4)$, $C(6,2)$, $D(0,-1)$. Which inequalities are satisfied by **all the points inside** the quadrilateral but **none of the points outside**?
4. Rotate by 90 counter-clockwise about the origin the lines given by equations
(a) $y=\frac{1}{2}x+1$;
(b) $y=-\frac{1}{4}x+\frac{5}{4}$;
The rotated lines should be perpendicular to the original lines; are they?