

**MATH 6 [2026 MAR 15]**  
**HANDOUT 19 : GEOMETRY WITH CARTESIAN COORDINATES II:**  
**LINE INTERSECTIONS AND GRAPHIC SOLUTIONS**

PERPENDICULAR LINES

In the previous class, we studied linear equations describing lines on a plane. The most general line equation is  $Mx + Ny = C$  and can describe any line (including horizontal and vertical). For example, equation  $3x + 2y = 1$  is neither horizontal or vertical, and its slope and intercept can be found by algebra

$$3x + 2y = 1 \iff y = -\frac{3}{2}x + \frac{1}{2} = kx + b,$$

so its slope is  $k = (-3/2)$  and its intercept with  $y$ -axis is at point  $(0, b) = (0, \frac{1}{2})$ .

Also, in the homework you might have observed that **two lines are perpendicular** if their slopes are related as  $k_1 k_2 = -1$ . A perpendicular line with the same intercept has equation

$$y = \frac{2}{3} + \frac{1}{2}.$$

Now we can write an equation for a **perpendicular bisector**.

For example, if  $A(1, 4)$ ,  $B(5, 2)$ , then

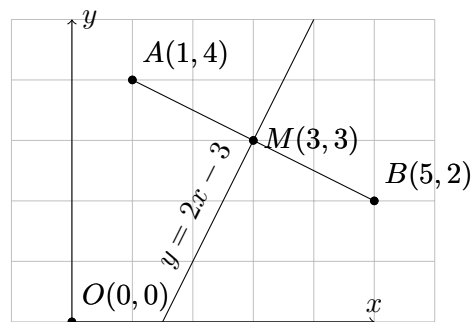
$$\text{line } (AB) \text{ slope } k = \frac{y_A - y_B}{x_A - x_B} = \frac{+2}{-4} = -\frac{1}{2},$$

$$\text{perpendicular slope } k_{\perp} = -1/k = +2,$$

$$\text{the midpoint is } M\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = M(3, 3),$$

$$[AB] \text{ bisector equation is } y - y_M = k_{\perp}(x - x_M)$$

$$\iff y - 3 = 2(x - 3) \iff y = 2x - 3.$$



You can check this on graphed paper!

LINES RELATIVE TO EACH OTHER

From geometry, we know that two lines on a plane may

- not intersect (be parallel to each other)
- intersect at a single point
- coincide (lie exactly on top of each other), or “intersect” at all points.

Here, “intersect” is in the sense we used when talking about sets: if set  $A = \{\forall(x, y) \in \text{line } a\}$  and  $B = \{\forall(x, y) \in \text{line } b\}$  then their intersection may be either one point, empty set ( $a \parallel b$ ), or their entire selves if lines  $a = b$ . All intersection points have to satisfy equations of both lines.

**Example** Given points  $A(1, 2)$ ,  $B(6, 1)$ ,  $C(5, 3)$ ,  $D(3, 4)$ , find intersection of lines  $(AB)$  and  $(CD)$

**Solution**

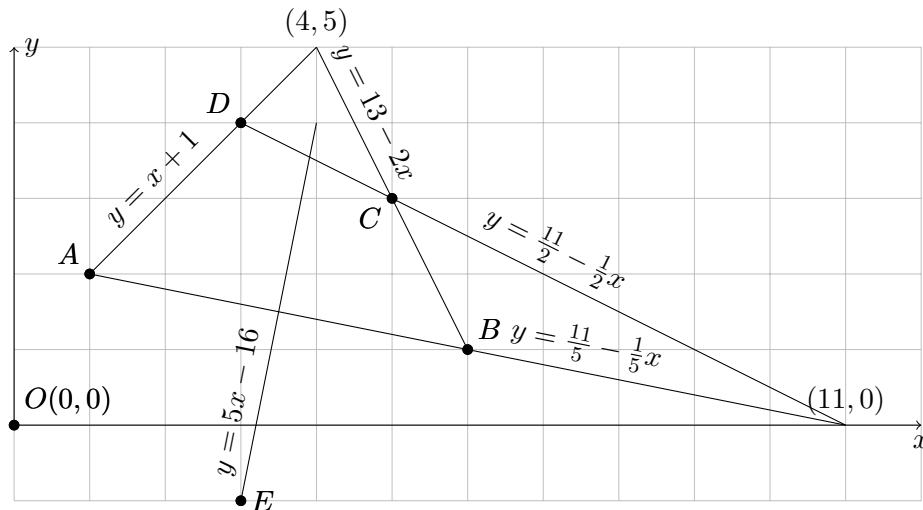
Line  $(AB)$  has slope  $(y_A - y_B)/(x_A - x_B) = -\frac{1}{5}$  and its equation is  $y - y_A = -\frac{1}{5}(x - x_A)$  or  $y - 2 = -\frac{1}{5}(x - 1)$ ; slope of line  $CD$  is  $(y_C - y_D)/(x_C - x_D) = -\frac{1}{2}$  and its equation is  $y - 4 = -\frac{1}{2}(x - 3)$ ; therefore

$$\text{line } (AB) = \{\forall(x, y) : y - 2 = -\frac{1}{5}(x - 1)\},$$

$$\text{line } (CD) = \{\forall(x, y) : y - 4 = -\frac{1}{2}(x - 3)\}.$$

Solve the 2-equation system to find the intersection  $(AB) \cap (CD)$ :

$$\begin{cases} y - 2 = -\frac{1}{5}(x - 1) \\ y - 4 = -\frac{1}{2}(x - 3) \end{cases} \iff \begin{cases} x = 11 - 5y \\ -2y + 8 = 11 - 5y - 3 \end{cases} \iff \begin{cases} x = 11 \\ y = 0 \end{cases}$$



We can also construct the altitude (the perpendicular segment to a line from a given point). For example, the line through  $E(3, -1)$  perpendicular to line  $(AB)$  above has slope  $k_{\perp(AB)} = +5$  and equation  $y + 1 = 5(x - 3)$  or  $y = 5x - 16$ . It intersects line  $(AB)$  at point

$$\begin{cases} y - 2 = -\frac{1}{5}(x - 1) \\ y + 1 = 5(x - 3) \end{cases} \Leftrightarrow \begin{cases} x = 11 - 5y \\ y + 1 = 55 - 25y - 15 \end{cases} \Leftrightarrow \begin{cases} x = 3.5 \\ y = 1.5 \end{cases}$$

#### CLASSWORK

1. Find the equation of the bisectors to segments  $[AB]$ ,  $[BC]$ ,  $[CD]$  if  $A(-2, 0)$ ,  $B(6, 2)$ ,  $C(0, 4)$ .
2. Side bisectors in a triangle intersect at one point. Check it is indeed the case for the triangle  $ABC$  above.
3. For the triangle  $ABC$  above, write the equations of the medians and check that they also intersect at one point.

#### HOMEWORK

1. Using coordinates of the points  $A(-3, 0)$ ,  $B(0, 1)$ ,  $C(2, 0)$ , and  $D(4, 4)$ , find the intersections of the lines  $(AB)$  and  $(CD)$ .
2. Find the equations of the perpendicular bisectors between points
  - (a)  $(0, 1)$  and  $(6, 3)$  ;
  - (b)  $(5, 1)$  and  $(1, 7)$  .
3. Find the equations of lines through point  $A$  and perpendicular to line  $(BC)$  if
  - (a)  $A(1, -1)$ ,  $B(-2, 1)$ , and  $C(1, 2)$  ;
  - (b)  $A(2, 1)$ ,  $B(0, -1)$ , and  $C(-1, 3)$  .
4. In addition to three perpendicular bisectors, three altitudes in a triangle also intersect at one point. Write equations of altitude lines in triangle  $ABC$ , where  $A(0, 0)$ ,  $B(6, 0)$ ,  $C(8, 4)$ , and check that they all intersect at the same point.