MATH 6 [2026 MAR 8]
HANDOUT 19 : GEOMETRY WITH CARTESIAN COORDINATES I

COORDINATE GEOMETRY: INTRODUCTION

In this section of the course we are going to study coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the x**-axis** and the y**-axis**. x-axis is usually drawn horizontally, and y-axis — vertically. These two axes have a **scale** – ticks with marked distance from the origin.

The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point P, draw lines through P perpendicular to the x- and y-axes. These lines intersect the axes in points with coordinates x_0 and y_0. Then the point P has x-coordinate x_0, and y-coordinate y_0, and denoted as $P(x_0, y_0)$.

Coordinates make some things very easy to do. For example, the **midpoint** M of segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The **length** of segment AB is found using Pythagorean theorem: $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

GRAPHS AND LINES

If you have an equation with variables x, y (for example, $x + 2y = 0$ or $y = x^2 + 1$), you can plot all points $M(x, y)$ whose coordinates satisfy this equation on the coordinate plane. All these points will usually make some smooth line or curve. This curve is called the **graph** of this equation.

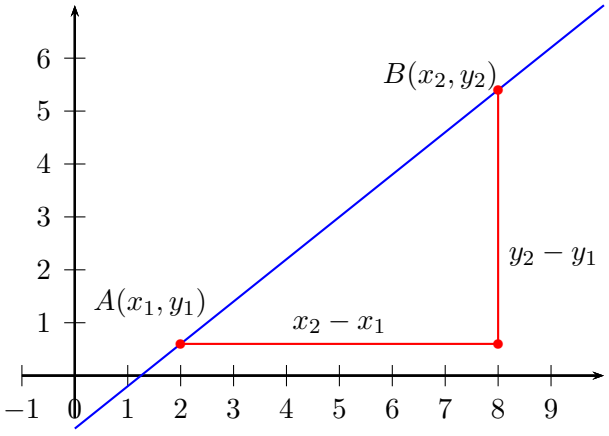
Any equation of the form (with some numbers m, b)

$$y = mx + b$$

has a graph that is a straight line. The slope of this line is determined by m: as you move along the line, y changes m times as fast as x; if you increase x by 1, then y will increase by m. If two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are on the line,

$$y_2 - y_1 = m(x_2 - x_1)\,.$$

For example, the line intersects the vertical axes ($x = 0$) at point $X(0, b)$; b is called the **intercept**.



Conversely, if two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are on the line, its **slope** can be computed by dividing change of y: $y_2 - y_1$ by the change of x: $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Two lines are **parallel** if and only if they have the **same slope**. A **horizontal** line has ZERO slope, and its equaiton is $y = b$; it is at distance $|b|$ from the horizontal axis and intersects the vertical axis at point $(0, b)$.
- A **vertical** line is a special case: its slope is undefined, and its equation is $x = a$ for some number a; it is at distance $|a|$ from the vertical axis and intersects the horizontal axis at point $(a, 0)$.
- The **most general** way to write the equation of a line is $Mx + Ny = C$, for some numbers M, N, and C.

CLASSWORK

1. Find the coordinates of
(a) point B which is 2 units below and 5 units to the right of point $A(3,4)$?
(b) point D which 4 units to the right and 3 units below of point $C(1,9)$?
What is the length of segment AC? of segment BD?

2. Find the coordinates of the midpoint of the segment AB, where $A=(-2,7)$, $B=(6,-5)$. Write the equation of the line (AB). Can you write the equation of the bisector?

3. Find k if $(1,9)$ is on the graph of $y-2x=k$. Sketch the graph.

4. Find k if $(1,k)$ is on the graph of $5x+4y-1=0$. Sketch the graph.

5. Write equations of lines going through points

(a) $(0,0)$ and $(4,3)$; (b) $(5,2)$ and $(-1,5)$; (c) $(0,-1)$ and $(4,7)$; (d) $(8,4)$ and $(6,0)$.

HOMEWORK

1. Find the coordinates of
(a) point B which is 2 units above and 5 units to the left of point $A(7,6)$?
(b) point D which 4 units to the right and 6 units below of point $C(-5,1)$?
What is the length of segment AC? of segment BD?

2. Find the coordinates of the midpoint of the segment AB, where $A=(3,11)$, $B=(7,5)$. Write the equation of the line (AB). $(^*)$ Can you write the equation of the bisector?

3. Draw points $A(4,1)$, $B(3,5)$, $C(-1,4)$. If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex? What is the area of this square?

4. $ABCD$ is a parallelogram (different in each part below); what are the coordinates of
(a) vertex D if the other three vertices are $A(0,0)$, $B(2,3)$, $C(4,1)$?
(b) vertex C if the other three vertices are $A(0,0)$, $B(1,3)$, $D(5,-2)$?
(c) vertex B if the other three vertices are $A(0,0)$, $C(1,5)$, $D(3,-2)$?
Can you guess the general rule: what are the coordinates of point D if the other three vertices are $A(0,0)$, $B(b_1,b_2)$, $C(d_1,d_2)$?

5. Consider the triangle ΔABC with the vertices $A(-2,-1)$, $B(2,0)$, $C(2,1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.

6. What is the slope of a line whose equation is $y=2x$? What is the slope of a line whose equation is $y=mx$?

7. In this problem you will find equations that describe some lines.
(a) What is the equation whose graph is the y-axis?
(b) What is the equation of a line whose points all lie 5 units above the x-axis?
(c) Is the graph of $y=x$ a line? Draw it.
(d) Find the equation of a line that contains the points $(1,-1)$, $(2,-2)$, and $(3,-3)$.

8. For each of the equations below, draw the graph, then draw the perpendicular line through the origin $(0,0)$ and then write the equation of the perpendicular line:

(a) $y=2x$ (b) $y=3x$ (c) $y=-x$ (d) $y=-\frac{1}{2}x$

$(^*)$ Can you guess the rule: if the slope of a line is k, what is the slope of a perpendicular line?