MATH 6 [2026 MAR 1]
HANDOUT 18 : REVIEW: SYSTEMS OF LINEAR EQUATIONS

SYSTEMS OF LINEAR EQUATIONS

Systems of linear equations are two or more linear equations that are using the same variables, hold true at the same time and have to solved together. Knowing how to solve systems of equations is useful especially when solving word problems. It is also very useful for doing geometry using coordinates, which will be the next topic.

The most general way to solve a system of two (or more) linear equations is the *SUBSTITUTION* method. Let's look at an example:

$$\begin{cases} x + \ \ y = 5 \\ 6x - 5y = -3 \end{cases}$$

Begin by taking the first equation $x + y = 5$ and "solving" it to find unknown variable x:

$$x = 5 - y\,.$$

This is not the solution yet – we do not know y to calculate x.
However, we can substitute $5 - y$ for x in the second equation:

$$6(5-y) - 5y = -3\,, \quad \Leftrightarrow \quad 30 - 6y - 5y = -3\,, \quad \Leftrightarrow \quad 11y = 30 + 3\,, \quad \Leftrightarrow \quad y = 3$$

(remember the *EQUIVALENCE* sign "⇔" from the logic chapter in the Fall? a chain of equivalent "⇔" equations mean they all have the same solution.)
Finally, to find x, substitute y in the expression for x:

$$x = 5 - 3 = 2$$

You can easily check that your answer (always a good idea) by substituting $x = 2$, $y = 3$ into the original equations.

This method works for more than two more than two equations (for example, three equations with three variables x, y, z). After substituting the first variable (x), you can do the same for the second variable (y) until you have just one equation with one unknown. This is why you must have at least the same number of equations as unknowns to have a unique (single) solution.

You can use equations and substitute variables in any order, but it helps to select the simpler equation first.

When your equations are particularly simple, it may be easier to use the *ADDITION* method, for example

$$\begin{cases} (1):\ x + y = 5 \\ (2):\ x - y = 1 \end{cases}$$

Because equations can be added and subtracted, we can quickly solve the system by

$$\begin{cases} (1)+(2): & 2x & = 5+1 \\ (1)-(2): & 2y & = 5-1 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x & = 3 \\ y & = 2 \end{cases}$$

In another example, you have to multiply equations by carefully chosen coefficients to cancel (eliminate) one of the variables:

$$\begin{cases} (1):\ 3x - 2y = 5 \\ (2):\ 2x + 4y = 14 \end{cases} \quad \Leftrightarrow \quad \begin{cases} 2(1)+(2): & 2*3x + 2x & = 2*5 + 14 \\ 3(2)-2(1): & 3*4y + 2*2y & = 3*14 - 2*5 \end{cases} \quad \Leftrightarrow \quad \begin{cases} 8x & = 24 \\ 16y & = 32 \end{cases}$$

CLASSWORK

1. Solve these equations by substitution:

(a) $\begin{cases} 4x + \ \ y = 5 \\ -2x + 3y = -13 \end{cases}$ (b) $\begin{cases} -5x + 2y = 5 \\ x + 7y = -1 \end{cases}$ (c) $\begin{cases} \frac{3}{2}x + \frac{1}{3}y = 8 \\ -\frac{3}{2}x + \frac{5}{3}y = 4 \end{cases}$

2. Solve these equations by addition:

(a) $\begin{cases} 3x + 8y = 15 \\ 2x - 8y = 10 \end{cases}$ (b) $\begin{cases} -5x + 8y = -18 \\ 5x + 2y = 58 \end{cases}$ (c) $\begin{cases} 5x - 3y = 26 \\ -5x - 2y = -16 \end{cases}$

3. Solve the following system of linear equations:
$$\begin{cases} 2(x+3(y+1)-1)=9 \\ 3(x-2(y+2)+1)=6 \end{cases}$$

4. A parking meter contains 27 coins consisting only of dimes and quarters. If the meter contains $4.35, how many of each type of coin is there?

5. The sum of two numbers is 27. Twice the larger number is 11 less than 3 times the smaller number. What are the two numbers?

6. A chemistry student needs 60 ml of a 26% salt solution. He has two salt solutions, A and B, to mix together to form the 60 ml solution. Salt solution A is 30% salt and salt solution B is 20% salt. How much of each solution should be used?

7. After paying a 30% tax on a property sold, three businessmen shared the amount left in the ratio $3:2:2$. If the businessman whose share was the largest received $21,000$\$, how much was the property worth before paying the tax?

Homework

(Problems with a star are optional)

1. Solve the following systems of linear equations:

(a) $\begin{cases} 5x+2y=16 \\ 2x+3y=13 \end{cases}$ (b) $\begin{cases} x-3y=17 \\ 8x+2y=46 \end{cases}$ (c) $\begin{cases} \frac{5}{6}x-\frac{9}{10}y=-2 \\ \frac{1}{3}x+\frac{2}{5}y=3 \end{cases}$

2. Solve these equations by addition:

(a) $\begin{cases} x-4y=-18 \\ -x+3y=11 \end{cases}$ (b) $\begin{cases} -3x+2y=56 \\ -5x-2y=24 \end{cases}$ (c) $\begin{cases} -9x+4y=6 \\ 9x+5y=-33 \end{cases}$

3. The sum of two numbers is $\frac{41}{35}$ and the difference is $\frac{1}{35}$. What are the two numbers?

4. A 2-digit number is larger by 9 than the number with the digits reversed. The sum of the digits is 7. Find the number. **Hint:** One can always write $54 = 5 \times 10 + 4$.

5. A play was attended by 342 people, some adults and some children. Admission for adults was $1.50 and for children 75¢. How many adults and how many children attended the play, if all the tickets sold for $354?

6. A bag contains only nickels and dimes. The value of the collection is $2. If there are 26 coins in all, how many of each coin are there?

***7.** A motor boat can travel 45 miles downstream in 3 hours and 22 miles upstream in 2 hours. Find is the speed of the boat in still water and find the speed of the current. (Hint: speed=distance/time)

***8.** A tank can be filled in 10 minutes from faucet A at a rate of 50ml/s. If another faucet B is turned on when the tank is one-third full, it will take another 4 minutes and 10 seconds to fill the tank. Find the flow of water from faucet B.