

**MATH 6 [2026 FEB 22]**  
**HANDOUT 17 : REVIEW: ARITHMETIC AND GEOMETRIC SEQUENCES**

ARITHMETIC SEQUENCES

A sequence of numbers is an **arithmetic sequence** or **arithmetic progression** if the difference between consecutive terms is the same number, the **common difference** or  $d$ .

We can find the  $n$ -th term if we know the 1st term and  $d$ ,

$$a_n = a_1 + (n - 1)d$$

The sum of an arithmetic sequence from  $a_1$  to  $a_n$  is

$$S_n = \frac{a_1 + a_n}{2} \cdot n$$

GEOMETRIC SEQUENCES

A sequence of numbers is a **geometric sequence** or **geometric progression** if the next number in the sequence is the current number times a fixed constant called the **common ratio** or  $q$ .

We can find the  $n$ -th term if we know the 1st term and  $q$ .

$$b_n = b_1 q^{n-1}$$

The sum of a geometric sequence from  $b_1$  to  $b_n$  is

$$S_n = b_1 + b_2 + \dots + b_n = b_1 \frac{q^n - 1}{q - 1} = \frac{qb_n - b_1}{q - 1}$$

If the common ratio is less than 1 in magnitude, the sum of the entire sequence is finite,

$$S_\infty = \frac{b_1}{1 - q}$$

REPEATED FRACTIONS

When dividing two integer numbers you often get a repeated decimal fraction, for example,

$$0.545454\dots = 0.(54)$$

(the digits in the parentheses are repeated indefinitely). There is a theorem saying that such repeated decimal fractions are rational numbers (can be written as a ratio of some integer numbers)

How do you convert a repeated fraction to a ratio? Note that this number is a sum of a geometric sequence,

$$0.(54) = 0.545454\dots = 0.54 + 0.0054 + 0.000054 + \dots$$

(what are the first term and the common ratio?) therefore its total sum is

$$0.(54) = \frac{0.54}{1 - 0.01} = \frac{54}{99} = \frac{6}{11}.$$

ZENO'S PARADOX OF ACHILLES AND THE TORTOISE

A legend says that Achilles, a Greek warrior, once was challenged by the Tortoise to a 100 meter race. Fleet-footed Achilles laughed and said he would certainly win, so the Tortoise asked for a head start of 90 meters. Achilles was sure he would still win but Tortoise convinced him otherwise by this argument:

- while Achilles runs for 90 meters to where the Tortoise started, it crawls 9 meters,
- when Achilles runs the next 9 meters, the Tortoise crawls for 0.9 meter more,
- and so on on and on, the Tortoise will always be ahead of Achilles.

Do you think Achilles will ever catch up with the Tortoise? If so, how far from the start?

### CLASSWORK

1. Determine if these sequences are arithmetic; if not, find the term which can be changed to make them arithmetic, and write the formulas for  $n$ -th term
  - (a) 101, 94.5, 87, 81.5, ... ;
  - (b) 23,  $29\frac{1}{3}$ , 3523, 41, ... .
2. Determine if each sequence is geometric. If not, find the term which can be changed to make it geometric. Find the common ratio.
  - (a) 7, 21, 63, 189, 567, 1701, ... ;
  - (b) 64, 16, 4, 1,  $\frac{1}{4}$ ,  $\frac{1}{6}$  ;
  - (c) 3, 16, 48, 196, ... .
3. Find the sum of these geometric sequences
  - (a)  $b_1 = \frac{1}{2}$  and  $b_8 = 64$  ;
  - (b)  $b_1 = 1$  and  $b_9 = \frac{1}{6}$  .
4. Find the sum of these geometric sequences
  - (a)  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$  ;
  - (b)  $9 + 3 + 1 + \dots$  .
5. Convert to a ratio of two integer numbers
  - (a) 0.189189189... ;
  - (b) 0.150150150... ;
  - (c) 0.51515151... .

### HOMEWORK

1. Write the formula describing the following arithmetic sequences :
  - (a) 6, 14, 22, 30, ... ;
  - (b) sequence with  $a_5 = -32$  and  $a_{18} = 85$ .
2. Calculate the sum of these arithmetic sequences beginning and ending with
  - (a)  $a_1 = 2$  and  $a_{10} = 13$  ;
  - (b)  $a_1 = 16$  and  $a_6 = \frac{3}{2}$  ;
  - (c)  $a_1 = 1$  and  $a_{100} = 2$  .
3. Write the formula describing the following geometric sequences :
  - (a) 64, 16, 4, 1,  $\frac{1}{4}$ ,  $\frac{1}{16}$ , ... .
  - (b) sequence with  $b_5 = -32$  and  $b_8 = 4$ .
4. Insert additional terms into the following sequence so it remains a geometric sequence
  - (a)  $\frac{1}{8}$ ,  $\frac{1}{2}$ , 2, 8, 32, ... ;
  - (b) 54, 18, 6, 2,  $\frac{2}{3}$ , ... ;
  - (c)  $62\frac{1}{2}$ , 25, 10, 4,  $\frac{8}{5}$ , ... .
5. Determine the common ratio  $q$  and calculate the sum of the geometric sequences beginning and ending with
  - (a)  $b_1 = \frac{1}{4}$  and  $b_5 = 4$  ;
  - (b)  $b_1 = 81$  and  $b_6 = -\frac{1}{3}$  ;
  - (c)  $b_1 = \frac{1}{3}$  and  $b_7 = 9$  .
6. Convert to a ratio of two integer numbers
  - (a) 0.297297297... .
  - (b) 0.15151515... .
  - (c) 0.72727272... .
  - (d) 0.168168168... .