MATH 6 [2025 NOV 9]
HANDOUT 8: SETS III. SUBSETS AND LOGIC EXPRESSIONS

SUBSETS

Logic operations and sets are closely related. It is most clear in writing sets by specifying some property

$$A = \Big\{\forall x \,|\, \text{some property of } x \,=\, \text{True}\Big\}$$

A "tautology" is saying that A consists of all x that belong to A, $x \in A$:

$$A = \Big\{\forall x \,|\, x \in A\Big\}$$

Then the relations between set unions, intersections, differences and complements of sets and logic operations OR , AND, AND NOT, NOT

- $A \cup B = \Big\{\forall x \,|\, (x \in A) \,\text{OR}\, (x \in B)\Big\}$;
- $A \cap B = \Big\{\forall x \,|\, (x \in A) \,\text{AND}\, (x \in B)\Big\}$;
- $A \backslash B = \Big\{\forall x \,|\, (x \in A) \,\text{AND}\, (\text{NOT}\, (x \in B))\Big\}$;
- $\overline{A} = \Big\{\forall x \,|\, \text{NOT}\, (x \in A)\Big\}$.

What is the equivalent of the implication, $(x \in A) \Longrightarrow (x \in B)$? It means that A is a subset of B:

$$A \subset B \quad \text{ if and only if } (x \in A) \Longrightarrow (x \in B)$$

Intervals. The following notations are used when we talk about intervals on the number line. Intervals may have end points included or excluded: [and] represent that the end point is included, while (and) indicate that the end point is excluded.

- $[a, b] = \{x \mid a \leq x \leq b\}$ is the interval from a to b (including endpoints),
- $(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
- $[a, \infty) = \{x \mid a \leq x\}$ is the half-line from a to infinity (including a),
- $(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)

Note that either infinity ($+\infty$ or $-\infty$) is NEVER included because it is not a number. Using union operation, we can have multiple intervals that are not overlapping, for example,

$$[0, 5] \backslash [1, 3] = [0, 1) \cup (3, 5] \,,$$
$$\overline{(1, 4)} = (-\infty, 1] \cup [4, \infty) \,.$$

CLASSWORK

1. Consider sets (intervals) $A = [2, 5)$, $B = (3, 7]$, $C = [4, 6]$. Draw sets A, B, C on a number axis. Then draw the following sets and write them as intervals or unions of intervals:

(a) $A \cap B$; (b) $A \cup B$; (c) $A \backslash C$; (d) $B \backslash C$; (e) $(A \cup B) \backslash C$.

2. On a Cartesian plane with axes (x, y), show the following sets:
(Hint: You can show included points with a solid line and excluded points with a dashed line)

(a) $A = \{\forall (x, y) \,|\, x > 0\}$;

(b) $B = \{\forall (x, y) \,|\, y \leq 0\}$;

(c) $C = \{\forall (x, y) \,|\, y \leq x - 2\}$;

(d) $A \cap B \cap C$.

HOMEWORK

1. Draw the following sets on the number line and describe in the interval notation:
   (a) Set of all numbers x satisfying $x \leq 3$ and $x > -10$;
   (b) Set of all numbers x satisfying $x \leq 3$ or $x < -10$
   (c) Set of all numbers x satisfying $x < -10$ or $x > 2$
   (d) Set of all numbers x satisfying $x > -10$ and $x \leq 2$

2. Let $A = [1, 3] = \{x \mid 1 \leq x \leq 3\}$, $B = \{x \mid x \geq 2\}$, $C = \{x \mid x \leq 1.5\}$. Draw on the number line the following sets: $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.

3. Let $A = \{1, 2, 3\}$. Find all the subsets of A. Do not forget the empty set $\emptyset$ and set A itself.

4. Find sets A, B, C if you know that $A \cup B = \{1, 3, 4, 5, 7\}$, $B \cup C = \{1, 2, 4, 5, 6, 8, 9\}$, $(A \cup B) \cap C = \emptyset$, $(B \cup C) \cap A = \{1, 5\}$.

5. Find A if you know that $A \cup \{5, 7\} = \{3, 5, 7, 8\}$, $A \cap \{1, 2, 5, 7\} = \{5, 7\}$.

6. A barber in a small town decides that he will shave all men who do not shave themselves (and only them). Should he shave himself? [Of course, the barber is a man.]

7. A local frog named Filo running for parliament tells the public, at the famous amphibian debate convention, that Filo will speak for all frogs who do not speak for themselves, and Filo will speak for no one else. Can you logically deduce if Filo will speak for Filo's own self?

***8.** 150 people at a Van Halen concert were asked if they knew how to play piano, drums or guitar.
   (a) 18 people could play none of these instruments.
   (b) 10 people could play all three of these instruments.
   (c) 77 people could play drums or guitar but could not play piano.
   (d) 73 people could play guitar.
   (e) 49 people could play at least two of these instruments.
   (f) 13 people could play piano and guitar but could not play drums.
   (g) 21 people could play piano and drums.

   How many people can play piano? drums?