

MATH 6 [2025 OCT 19]
HANDOUT 5: LOGIC IV. CONDITIONS: NECESSARY AND SUFFICIENT

Today we continue writing formal logic expressions, or logic “formulas”.

CLASSWORK

In logic we operate with statements instead of numbers like in algebra. Just like in algebra, you can sometimes check yourself by picking example statements, such as

- A* This animal is a lynx,
- B* This animal is a type of cat,
- C* This animal is a mammal,
- D* This animal has pointy ears.
- E* This animal has fur.



- $A \implies B$ expresses a rule that *A* is **sufficient** for *B*:

“If this animal is a lynx, it is a type of cat”.

If you know that *A* is **True**, and the rule $A \implies B$ is correct, then *B* must also be **True**.

As a logic formula, $A \text{ AND } (A \implies B) = B$. Recall now that $(A \implies B)$ is the same as $(\text{NOT } A) \text{ OR } B$:

$$A \text{ AND } ((\text{NOT } A) \text{ OR } B) = (A \text{ AND } (\text{NOT } A)) \text{ OR } (A \text{ AND } B) = \text{False OR } (A \text{ AND } B) = (A \text{ AND } B)$$

therefore both *A* and *B* must be **True**.

Let’s use logic formulas to show the following:

- An equivalent rule is $(\text{NOT } B) \implies (\text{NOT } A)$, or that *B* is **necessary** for *A*

“If this animal is not a type of cat, it is not a lynx”.

If you know that *B* is **False** then *A* must be **False**.

- (Syllogism) two rules combined together $(A \implies B) \text{ AND } (B \implies C)$ imply that $(A \implies C)$

“If this animal is a lynx, ($\implies$) it is a type of cat; a cat ($\implies$) is a type of mammal.

Therefore, a lynx ($\implies$) is a type of mammal.”

Can you prove that $((A \implies B) \text{ AND } (B \implies C)) \implies (A \implies C)$ is **True** for any *A, B, C*?

Could the reverse, $(A \implies C) \implies ((A \implies B) \text{ AND } (B \implies C))$, likewise always be **True**?

Other exercises:

1. Above we showed that $X \text{ AND } ((\text{NOT } X) \text{ OR } Y)$ is equivalent to $X \text{ AND } Y$.
What is $X \text{ OR } ((\text{NOT } X) \text{ AND } Y)$ equivalent to?
2. $X \iff Y$: both must be True, or both must be **False** ;
X is **necessary and sufficient** for *Y*, and vice versa. Can you give think of an example?
Make truth tables for $(X \iff Y)$, and $(X \text{ XOR } Y)$. How do these expressions compare?
3. What is the logic formula for *W* that matches this truth table?

<i>X</i>	<i>Y</i>	<i>Z</i>	<i>W</i>
T	T	F	T
T	F	T	T
F	F	T	T
(all else)			F

HOMEWORK

1. Many trucks carry the message: “If you do not see my mirrors, then I do not see you”. Can you rewrite it in an equivalent form without using the word “not”?
2. If today is Thursday, then Jane’s class has library day. If Jane’s class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,...
3. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore, ...
4. Check that $\text{NOT}(A \implies B)$ is equivalent to $A \text{ AND } (\text{NOT } B)$.
What is the equivalent of saying “It is not true that if an animal is a bird, it can fly”?
5. Are formulas $A \implies B$ and $B \implies A$ equivalent? Are formulas $A \implies B$ and $\text{NOT } A \implies \text{NOT } B$ equivalent? Check this by writing the truth tables for each of them.
6. A mom tells her son “If you do not do the dishes, you will not go to the movie”. Is it the same as “If you do the dishes, you go to the movie?”
7. A teacher tell the student “If you do not take the final exam, you get an F”. Does it mean that
 - (a) If the student does take the final exam, he will not get an F
 - (b) If the student does not get an F, it means he must have taken the final exam.
8. Here is another one of Lewis Carrol’s puzzles.
 - (a) All hummingbirds are richly colored.
 - (b) No large birds live on honey.
 - (c) Birds that do not live on honey are dull in color.Therefore, ...
9. And another one:
 - (a) My saucepans are the only things I have that are made of tin.
 - (b) I find all your presents very useful.
 - (c) None of my saucepans are of the slightest use.Therefore, ...
10. On the island next to he island of knights and knaves there are 3 kinds of people:
 - knights, who always tell the truth,
 - knaves, who always lie, and
 - normal people, who sometimes lie and sometimes tell the truthOn that island, you meet 3 people, A, B, and C, one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements:
A : I am normal
B : That is true
C : I am not normal
What are A, B, and C?