1. CLASSWORK

Truth tables can be used to show that two logic expressions are equivalent.
However, they become too long with more than 2 statements: you need to consider a table with 8 lines for 3 statements, 16 lines for 4 statements, etc. This is where we can use logic formulas using AND, OR, XOR, NOT, $\Longrightarrow$.

- You can combine more than two statements using AND: $(A \text{ AND } B \text{ AND } C)$ is True if **all of** A, B, and C are True, and False otherwise.
- The same goes for OR, but now $(A \text{ OR } B \text{ OR } C)$ is True if **ANY of** A, B, and C is True, and False only if they are all False
- Double negative : $\text{NOT}(\text{NOT } A) = A$; you can also use over-line to indicate NOT: $\text{NOT } A = \overline{A}$.

Operations AND, OR are similar to multiplication and addition: they are commutative like $a + b = b + a$ and $a * b = b * a$,
$$A \text{ AND } B = B \text{ AND } A, \quad A \text{ OR } B = B \text{ OR } A,$$
and associative like $(a + b) + c = a + (b + c)$
$$(A \text{ AND } B) \text{ AND } C = A \text{ AND}(B \text{ AND } C) = A \text{ AND } B \text{ AND } C,$$
$$(A \text{ OR } B) \text{ OR } C = A \text{ OR}(B \text{ OR } C) = A \text{ OR } B \text{ OR } C,$$
so if there are only AND's or only OR's in expressions, you do not need parentheses.

However, if you have both AND and OR, be sure to use parentheses because distributive rules apply like with multiplication and addition in algebra
$$A \text{ AND}(B \text{ OR } C) = (A \text{ AND } B) \text{ OR}(A \text{ AND } C),$$

similar to $a * (b + c) = a * b + a * c$.
There is also a rule in logic
$$A \text{ OR}(B \text{ AND } C) = (A \text{ OR } B) \text{ AND}(A \text{ OR } C),$$

which is unlike in algebra, where a similar rule does not work : $a + b * c \neq (a + b) * (a + c)$!
Exercises

1. What is the value of $A \text{ AND } True$? $A \text{ AND } A$? $A \text{ AND}(\text{NOT } A)$?
2. What is the value $A \text{ OR } True$? $A \text{ OR } A$? $A \text{ OR}(\text{NOT } A)$?
3. Is XOR associative, $(A \text{ XOR } B) \text{ XOR } C \overset{?}{=} A \text{ XOR}(B \text{ XOR } C)$?
4. (de Morgan rule) Can you express $\text{NOT}(A \text{ AND } B)$ using OR ? Can you express $\text{NOT}(A \text{ OR } B)$ using AND?

1. An inhabitant, Carl, of the island Knights and Knaves tells you "If Sue is a knave, then this road leads to the capital." However, later you learn that Carl is a knave. What does it tell you?

2. On the island of Knights and Knaves, a traveler meets two inhabitants: Carl and Bill. Bill says: "Carl is a Knave". Carl says: "If Bill is a Knight, then I am a Knight, too."
   Can you determine whether each of them is a Knight or a Knave?

3. Do statements "if you do not clean up your room, you will be punished" "clean up your room, or you will be punished" logically the same? Can you write the logic formulas for these statements?

4. Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)?
   (a) $(A \text{ OR } B) \text{ AND} (A \text{ OR } C)$
   (b) $A \text{ OR} (B \text{ AND } C)$.

5. We have defined a logical operation, XOR (exclusive or) as follows: A XOR is true if exactly one of A, B is true, and false otherwise. Can you express XOR using only AND, OR, and NOT?

6. On the island of knights and knaves, you meet two inhabitants, Alex and Bruce. Alex says, "Bruce is a knave". Bruce says, "Alex is a knave". Who is a knave and who is a knight?

7. Write truth tables for formulas $A \text{ AND} (B \text{ OR } C)$ and $(A \text{ AND } B) \text{ OR } C$ (hint: there will be 8 rows in the table). Are these formulas equivalent (i.e., do they always give the same answer)?

8. The waiter in a restaurant tells you: "our fixed price dinner includes soup and appetizer or salad". Denoting
   - $A =$ your dinner will include soup,
   - $B =$ your dinner will include appetizer,
   - $C =$ your dinner will include salad,

   what would be the correct way to write his statement using letters A, B, C and logical operations AND, OR?

9. You probably know Lewis Carroll as the author of *Alice in Wonderland* and other books. What you might not know is that he was also a mathematician very much interested in logic, and had invented a number of logic puzzles. Here is one of them:
   You are given 3 statements.
   - (a) All babies are illogical.
   - (b) Nobody is despised who can manage a crocodile.
   - (c) Illogical persons are despised. Can you guess what would be the natural conclusion from these 3 statements?