MATH 6 [2025 SEP 28]
HANDOUT 3: LOGIC II. LOGIC OPERATIONS

CLASSWORK

Last time, we discussed the meaning of words "AND" and "OR" in mathematical logic. We use them all the time to combine simple statements into longer sentences to declare ideas, consequences, and explanations. Take three statements, any of which can be True or False on a given day:

- (A) "Sky is clear";
- (B) "Air is warm";
- (C) "School has a day off";
- (D) "We are going to a Halloween parade";
- (E) "It is raining outside";
- (F) "I need an umbrella";
- (G) "We are going to see a movie tonight".

Now, what makes a good day?
If you want to spend the day outside, it's good to have clear sky (A=True) and warm air (B=True)

A	B	good day? (A AND B)
T	T	T
T	F	F
F	T	F
F	F	F

C	D	good day? (C OR D)
T	T	T
T	F	T
F	T	T
F	F	F

D	G	D XOR G
T	T	F
T	F	T
F	T	T
F	F	F

On the other hand, a day off school or a Halloween parade will make a good day too. This is called **"inclusive OR"** because both can be True on a good day. It is different from **"exclusive OR"** (XOR), which is True when only one of the statements is True and the other is False. For example, you cannot go both to a movie and a Halloween parade in the evening, so you can say D XOR G is True (unless you are grounded and and cannot go to either).

Sometimes one has to invert (negate) a statement, so that it becomes its opposite. This is written as NOT, for example NOT E is True when it is NOT raining outside.

A special relation between two logic statements is when **one means that the other has to be True** (the second follows from the first). For example, "IF E (it is raining outside), THEN F (I need an umbrella)". On the other hand, if it is not raining outside (NOT E), you may still need an umbrella (perhaps you go on a long trip and expect that it may rain later). This relation between E and F is False only in the case when the condition E is true ("it is raining"), but the consequence is not (NOT F, "I do not need and umbrella"):

E (it is raining)	NOT E (not raining)
T	F
F	T

E	F	IF E THEN F	(NOT E) OR F
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

In this case they say "E is a sufficient condition to F: if E is True, F cannot be Fals. From the truth table, you can see that such relation between E and F is **equivalent** to (NOT E) OR F. A truth table is a common way to prove that logic statements are equivalent, although it is somewhat long.

Which of the statements A, B, C, D, E, F, G might be a sufficient condition to another?

HOMEWORK

1. Solve the following equations:

(a) $2x - 22 = 3(1 - x)$ (b) $1 - \frac{2}{7}x = \frac{1}{7}x$ (c) $1 - 8(1 - x) = 7x - 8$

2. On the island of Knights and Knaves, you meet two inhabitants: Carl and Bill. Carl says, "I and Bill are both knights or both knaves." Bill claims, "Only a knave would say that Carl is a knave." Can you figure out who is who? [Hint: first, rewrite Bill's claim in an easier to understand form.]

3. On the island of Knights and Knaves, you meet three inhabitants: Bob, Mel and Peggy. Bob says that it's not true that Peggy is a knave. Mel says that Peggy is a knight or Bob is a knave. Peggy claims, "Both I am a knight and Bob is a knave."

4. On the island of Knights and Knaves, you meet three inhabitants:Bozo, Carl and Joe. Bozo says that Carl is a knave. Carl tells you, 'Of Joe and I, exactly one is a knight.' Joe claims, 'Bozo and I are different.' Can you figure out who is who?

5. Using truth tables, show that these statements are equivalent

X	Y	NOT$(X$ OR $Y)$	(NOT X) AND (NOT Y)
T	T		
T	F		
F	T		
F	F		

X	Y	NOT$(X$ AND $Y)$	(NOT X) OR (NOT Y)
T	T		
T	F		
F	T		
F	F		

6. Once there lived a king who had a beautiful princess daughter. One day a knight came to the king who wanted to marry the princess. The king decided to test the knight's wits and he offered him a challenge. He showed the knight three doors. Behind one of the doors there was a princess. Behind another there was a hungry tiger. The room behind the third door was empty.

On each door there was a sign: the sign on the princess's room door was true, the sign on the tiger's room door was false, and the sign on the empty room door could be either false or true:

Room I	Room II	Room III
Room III is empty	The tiger is in room I	This room is empty

Which door should the knight open?

7. A certain convention numbered 100 politicians. Each politician was either crooked or honest. We are given the following two facts:

- At least one of the politicians was honest.
- Given any two of the politicians, at least one of the two was crooked.

Can it be determined from these two facts how many of the politicians were honest and how many of them were crooked?