MATH 6 [2025 SEP 21]
HANDOUT 2: LOGIC I. KNIGHTS AND KNAVES

Many of the questions of this assignment refer to the famous (among logic puzzle fans) island of Knights and Knaves. On this island, there are two kinds of people: Knights, who always tell the truth(T), and Knaves, who always lie and what they say is false(F). Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave

For example,

• *Peggy tells you that "of Zippy and I, exactly one is a knight".*
• *Zippy tells you that Peggy is a knight.*

So what do you do? Try to consider all possible combinations of people being either Knights and Knaves: Are any combinations are compatible with their words? Just one? More than one? This is called a **truth table**:

Peggy is	Zippy is	Peggy said	Zippy said	Could Peggy say it?	Could Zippy say it?	Possible?
Knight(T)	Knight(T)	✘(F)	✓(T)	✘(F)	✓(T)	✘(F)
Knave(F)	Knight(T)	✓(T)	✘(F)	✘(F)	✘(F)	✘(F)
Knight(T)	Knave(F)	✓(T)	✓(T)	✓(T)	✘(F)	✘(F)
Knave(F)	Knave(F)	✘(F)	✘(F)	✓(T)	✓(T)	✓(T)

Therefore, they could have said that only if they are both Knaves.

CLASSWORK

Can you determine who is a knight and who is a knave? Read carefully what they say!

- *'X' and 'Y'* means that both 'X' and 'Y' must be true, so neither may be false.
- *'X' or 'Y'* means that 'X', 'Y', or both may be true, but at least one must be true.
- *Either 'X' or 'Y'*, only one must be true ("XOR"), and the other must be false; this is often called exclusive-OR (XOR).

1. Sally claims, 'Zippy and I are not the same.' Zippy says, 'Of I and Sally, exactly one is a knight.'
2. Marge says, 'Zoey and I are both knights or both knaves.' Zoey claims, 'Marge and I are the same.'
3. Mel tells you, 'Either Ted is a knight or I am a knight.' Ted tells you that Mel is a knave.
4. Ted says, 'Of Zippy and I, exactly one is a knight.' Zippy says that Ted is a knave.
5. Alice claims, "Charlie could tell you that I am a knight." Brian says, "Either Alice is a knave, or I am a knight." Charlie says that the others are either both knaves or both knights. What are Alice, Brian, and Charlie?

Review: power properties

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$$

$$a^n \cdot a^m = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}} \underbrace{a \cdot a \cdot \dots \cdot a}_{m \text{ times}} = \underbrace{a \cdot a \cdot \dots \cdot a}_{(n+m) \text{ times}} = a^{n+m}$$

$$(ab)^n = \underbrace{ab \cdot ab \cdot \dots \cdot ab}_{n \text{ times}} = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}} \underbrace{b \cdot b \cdot \dots \cdot b}_{n \text{ times}} = a^n \cdot b^n$$

$$a^{-1} = \frac{1}{a}, \quad a^{-n} = \frac{1}{a^n}$$

$$\frac{a^n}{a^m} = \frac{\overbrace{a \cdot a \cdot \dots \cdot a}^{n \text{ times}}}{\underbrace{a \cdot a \cdot \dots \cdot a}_{m \text{ times}}} = a^{n-m} = \frac{1}{a^{m-n}}$$

Simplify the following expressions:

(a) $\frac{7^4 \cdot 11^2}{11^{-5} \cdot 7^2}$ (b) $\frac{42^2}{6^2}$ (c) $\frac{3^5 \cdot 3^{-4}}{3^9}$ (d) $\frac{(x^2yz^3)^2y^3z^2}{z^4y(xz)^3}$ (e) $\frac{(a^2b)^3(b^2c)^4}{((ab)^2c)^3}$

HOMEWORK

1. Find the greatest common divisor and least common multiple of 132 and 90.

2. Solve the following equation: $5 - 2(1 - x) = 9$.

3. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, "Neither Zoey nor I are knaves." So who is a knight and who is a knave?

4. On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "I and Sue are knights." So who is a knight and who is a knave?

5. On the island of knights and knaves, you meet two inhabitants: Bart and Ted. Bart claims, "I and Ted are both knights or both knaves." Ted tells you, "Bart would tell you that I am a knave." So who is a knight and who is a knave?

6. You meet two inhabitants: Betty and Peggy. Betty tells you that Peggy is a knave. Peggy tells you, 'Betty and I are both knights.'

Can you determine who is a knight and who is a knave?

7. You meet two inhabitants: Zed and Peggy. Zed says that Peggy is a knave. Peggy tells you, 'Either Zed is a knight or I am a knight.'

Can you determine who is a knight and who is a knave?

8. You meet two inhabitants: Zed and Alice. Zed tells you, 'Alice could say that I am a knight.' Alice claims, 'It's not the case that Zed is a knave.'

Can you determine who is a knight and who is a knave?

9. Simplify these expressions and show the answer in the exponent form

(a) $\frac{3^7 \cdot 2^7}{2^3 \cdot 2^4}$; (c) $\frac{7^9 \cdot 2^5}{7^2 \cdot 2^4}$; (e) $\frac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4}$;

(b) $\frac{6^5 \cdot 2^4}{3^5 \cdot 2^2}$; (d) $\frac{11^4}{11^2 \cdot 5^2 \cdot 5^3}$; (f) $\frac{x^2 \cdot y^2 \cdot x^{-3}}{x^2}$.