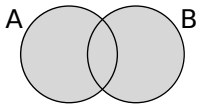
Sets, Part II

Set Operations

There are several operations that give new sets out of the old ones:

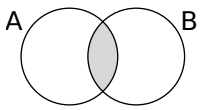
- $A \cup B$: *union* of A and B. It consists of all elements which are in either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$



- $A \cap B$: *intersection* of A and B. It consists of all elements which are in both A and B:

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$



- $\overline{A}$: *complement* of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.

Intervals

The following notations are used when we talk about intervals on the number line. Intervals may have end points included or excluded: $[$ and $]$ represent that the end point is included, while $($ and $)$ indicate that the end point is excluded.

$[a, b] = \{x \mid a \leq x \leq b\}$ is the interval from a to b (including endpoints),

$(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),

$[a, \infty) = \{x \mid a \leq x\}$ is the half-line from a to infinity (including a),

$(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)

Counting

We denote by $|A|$ the number of elements in a set A (if this set is finite). For example, if $A = \{a, b, c, \ldots, z\}$ is the set of all letters of English alphabet, then $|A| = 26$.

If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$: if there are 13 girls and 15 boys in the class, then the total is 28.

If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

(see problem 8 on HW).

Product rule

If we need to choose a pair of values, and there are a ways to choose the first value and b ways to choose the second, then there are ab ways to choose the pair.

For example, a position on a chessboard is described by a pair like a4; there are 8 possible choices for the letter, and 8 possible choices for the digit, so there are $8 \times 8 = 64$ possible positions.

It works similar for triples, quadruples, ... For example, if we toss a coin, there are 2 possible outcomes, heads (H) or tails (T). If we toss a coin 4 times, the result can be written by a sequence of four letters, e.g. HTHH; since there are 2 possibilities for each of the letters, we get $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible sequences we can get.