Truth Tables and Logic Laws

If

In addition to NOT, AND, and OR, there is also IF:
IF: when someone says "if A then B", and A is false, do you think he lied? for example, is the statement

"if sky is green, then 2+2=5"

true? The usual convention in mathematics is that *it is true*: any statement beginning with the words "if A then.. " is taken to be true in the case when A is false.

Truth tables

Logical variables take value True (T) or False (F).

Basic logic operations

NOT (for example, $\text{NOT } A$): true if A is false, and false if A is true.

AND (for example $A \text{ AND } B$): true if both A, B are true, and false otherwise

OR (for example $A \text{ OR } B$): true if at least one of A, B is true, and false otherwise. Sometimes also called "inclusive or" to distinguish it from the "exclusive or" described below

IF (as in "if A, then B; written $A \implies B$): if A is false, automatically true; if A is true, it is true only when B is true

As in usual algebra, logic operations can be combined, e.g. $(A \text{ OR } B) \text{ AND } C$.

Truth tables If we have a logical formula involving variables $A, B, C, \ldots$, we can make a table listing, for every possible combination of values of $A, B, \ldots$, the value of our formula. For example, the following is the truth tables for OR and IF:

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

Truth tables are useful in solving the problems about knights and knaves. Here is a typical problem: on the island of knights and knaves you meet two inhabitants, Zed and Alice. Zed tells you, 'I am a knight or Alice is a knave.' Alice tells you, 'Of Zed and I, exactly one is a knight.' We could solve it by making the following table:

Zed	Alice	Z is knight or A is knave	Of Z and A, exactly one is knight
knight	knight	T	F
knight	knave	T	T
knave	knight	F	T
knave	knave	T	F

Logic laws

We can combine logic operations, creating more complicated expressions such as A AND$(B$ OR $C)$. As in arithmetic, these operations satisfy some laws: for example A OR B is the same as B OR A. Here are two other laws:

$$\text{NOT}(A \text{ AND } B) \text{ is the same as } (\text{NOT } A) \text{ OR}(\text{NOT } B)$$
$$A \implies B \text{ is the same as } (\text{NOT } B) \implies (\text{NOT } A)$$

Truth tables provide the easiest way to prove complicated logical rules: if we want to prove that two formulas are equivalent (i.e., always give the same answer), make a truth table for each of them, and if the tables coincide, they are equivalent.