MATH 6: HANDOUT 27
FACTORIZATION. DIVISION OF EXPRESSIONS.

FACTORIZATION

When handling with large algebraic expressions, it is often possible to simplify them. One of of doing this is by **factorization**. As its name suggests, this method consists of finding a common factor in two or more terms. For example, in the following expression

$$7x + 9x - 5x$$

the three terms share the common factor x. Therefore, we can rewrite this expression as:

$$7x + 9x - 5x = (7 + 9 - 5)x = 11x.$$

In general, we will have the following identities:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

$$ab + ac = a(b + c)$$

POLYNOMIAL DIVISION

In arithmetics, we divide numbers using long division. We can do the similar process for dividing algebraic expressions.

For example, we can divide $2x^2 - 7x + 3$ by $x - 3$

$$\begin{array}{r|l} & 2x - 1 \\ \hline x - 3 & 2x^2 - 7x + 3 \\ & \underline{-2x^2 + 6x} \\ & \quad -x + 3 \\ & \quad \underline{x - 3} \\ & \quad\quad 0 \end{array}$$

This means that

$$2x^2 - 7x + 3 = (x - 3)(2x - 1)$$

Now, if we want to solve an equation

$$2x^2 - 7x + 3 = 0$$

we can equate both factors to 0, and get two solutions: when $x - 3 = 0$, then $x = 3$, and when $2x - 1 = 0$, then $x = 1/2$.

This can be useful if we want to solve an equation for which we don't have any good method. For example, if we are trying to solve an equation

$$2x^2 + 7x - 22 = 0$$

we can try to guess one of the solution using guess-and-check method. When we check small whole numbers, we can see that $x = 2$ is a solution, and that would mean that we can divide $2x^2 + 7x - 22$ by $x - 2$:

$$
\begin{array}{r|l}
 & 2x+11 \\
\hline
x-2 & 2x^2+7x-22 \\
 & \underline{-2x^2+4x} \\
 & 11x-22 \\
 & \underline{-11x+22} \\
 & 0
\end{array}
$$

This means that

$$2x^2 + 7x - 22 = (x-2)(2x+11)$$

and to solve this equation we have to equate two factors to 0: $x - 2 = 0$ gives us $x = 2$, and $2x + 11 = 0$ gives us $x = -5.5$.

Note that as with division of numbers, in some cases you may end up with a remainder:

$$
\begin{array}{r|l}
 & 2x+15 \\
\hline
x-3 & 2x^2+9x-22 \\
 & \underline{-2x^2+6x} \\
 & 15x-22 \\
 & \underline{-15x+45} \\
 & 23
\end{array}
$$