MATH 6: HANDOUT 25: SYSTEMS OF LINEAR EQUATIONS

SYSTEMS OF LINEAR EQUATIONS

Systems of linear equations are two or more linear equations that are using the same variables, hold true at the same time and have to solved together. Knowing how to solve systems of equations is useful especially when solving word problems. We are going to learn today how to solve a system of two linear equations using the substitution method and practice solving word problems.

When using the substitution method, select the simpler equation and try to express one variable in terms of the other variable, then substitute the variable in the other equation. You end up with an equation in one variable that you already know how to solve. Let's look at an example:

$$\begin{cases} 6x - 5y = -3 \\ x + \ \ y = 5 \end{cases}$$

Let's choose the second equation $x + y = 5$, $x = 5 - y$. Substitute x with $5 - y$ in the first equation. $6(5 - y) - 5y = -3$, $30 - 6y - 5y = -3$, $11y = 30 + 3$, $y = 3$. To find x, substitute y in the second equation. $x = 5 - 3 = 2$. You can always check that your answer is correct by plugging in 2 for x and 3 for y in the original two equations.

ARCHIMEDES' LAW

Recall the legend of Archimedes running naked through Syracuse shouting "Eureka, eureka!"

"Here is how it was. The Syracuse tyrant Hieron obtained from a goldsmith a golden crown and he wanted to check whether the goldsmith was not mixing silver into the gold. It was necessary to compare the volumes of the crown and a piece of pure gold of the same weight. Archimedes, by sinking into a bath flooding to the edges and seeing how the water displaced by his body overflowed the edges, suddenly realized that it is easy to measure the volumes of two bodies of different shapes."

Here is a more advanced version. The goldsmith was given gold and silver separately which were alloyed together to form a solid object (crown). Archimedes is required to find out whether or not the goldsmith had replaced part of the gold with silver. Let x_1, x_2 be the amounts of gold and silver in the ready-made crown. Its entire weight $x_1 + x_2 = W$ was easy to measure and verify that it was equal to the total weight given to the goldsmith. The goldsmith is no fool and this equality is found to be true. We now suspend the crown on a spring balance and immerse it in a full bath. Collecting the overflowed water we measure its volume V and weight P; we also read off the new reading (on the spring balance) of the weight B of the immersed crown. We find that $W - B = P$, that is, a body immersed in water loses the same amount of weight as the weight of the water displaced by it. This is, in fact, ARCHIMEDES'S LAW!

How do we determine the exact amounts of gold and silver in the alloy? The densities ρ_1, ρ_2 (masses per unit volume in terms of the weight on Earth) of our precious metals have been known for a long time. Gold is more dense than silver, so $\rho_1 > \rho_2$. Then the quantities $\frac{x_1}{\rho_1} = V_1$, $\frac{x_2}{\rho_2} = V_2$ give the volumes of each metal in the crown. Hence, $V_1 + V_2 = V$. Thus we have the relations

$$x_1 + x_2 = W \qquad \frac{x_1}{\rho_1} + \frac{x_2}{\rho_2} = V.$$

Thus we find the weights of

$$x_1 = \frac{\rho_1}{\rho_1 - \rho_2}(W - V\rho_2), \qquad x_2 = -\frac{\rho_2}{\rho_1 - \rho_2}(W - V\rho_1).$$