Coordinate Geometry: Introduction

In this section of the course we are going to study coordinate geometry. The basic notion is the coordinate plane – a plane with a given fixed point, called the origin, as well as two perpendicular lines – axes, called the x-axis and the y-axis. x-axis is usually drawn horizontally, and y-axis — vertically. These two axes have a scale – "distance" from the origin.

The scales on the axes allow us to describe any point on the plane by its coordinates. To find coordinates of a point P, draw lines through P perpendicular to the x- and y-axes. These lines intersect the axes in points with coordinates x_0 and y_0. Then the point P has x-coordinate x_0, and y-coordinate y_0, and the notation for that is: $P(x_0, y_0)$.

The midpoint M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Lines

Given some relation which involves variables x, y (such as $x + 2y = 0$ or $y = x^2 + 1$), we can plot on the coordinate plane all points $M(x, y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the graph of the given relation.

Every relation (equation) of the form:

$$y = mx + b$$

where m, b are some numbers, defines a straight line. The slope of this line is determined by m: as you move along the line, y changes m times as fast as x, so if you increase x by 1, then y will increase by m:
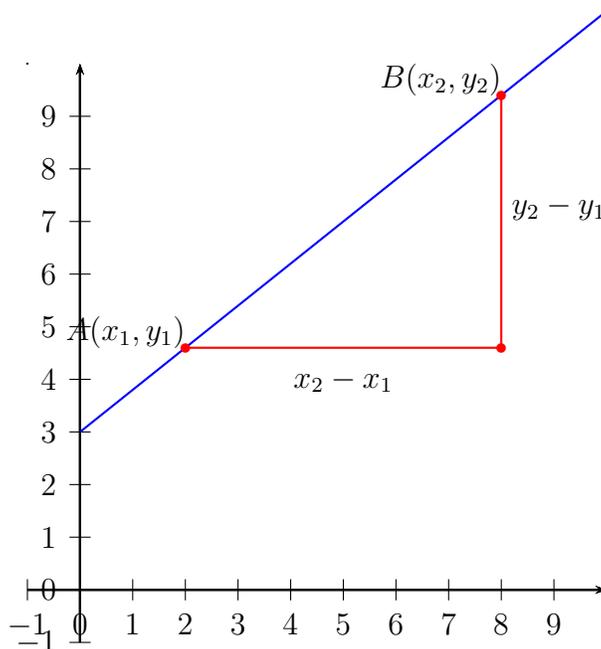
In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ slope can be computed by dividing change of y: $y_2 - y_1$ by the change of x: $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two non vertical lines are parallel if and only if they have the same slope.

In the equation $y = mx + b$, b is a y-intercept, and determines where the line intersects the vertical axis (y-axis).

The equation of the vertical line is $x = k$, and the equation of the horizontal line is $y = k$. Notice that in case of the vertical line, the slope is undefined.