MATH 6: HANDOUT 11
GEOMETRY: RULER AND COMPASS CONSTRUCTIONS

Constructions with ruler and compass

For the next couple of classes we will be mostly interested in doing the geometric constructions with ruler and compass. Note that the ruler can only be used for drawing straight lines through two points, not for measuring distances! When doing these problems, we need:

- Give a recipe for constructing the required figure using only ruler and compass
- **Explain why our recipe does give the correct answer**

For the first part, our recipe can use only the following operations:

- Draw a line through two given points
- Draw a circle with center at a given point and given radius
- Find and label on the figure intersection points of already constructed lines and circles.

For the second part, we will frequently use the results below.

Congruence tests for triangles

Recall that by definition, to check that two two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do with fewer checks.

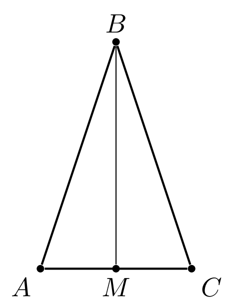
Axiom 1. *(SSS Rule). If $AB = A'B', BC = B'C'$ and $AC = A'C'$ then $\triangle ABC \cong \triangle A'B'C'$.*

Axiom 2. *(ASA Rule). If $\angle A = \angle A', \angle B = \angle B'$ and $AB = A'B'$ then $\triangle ABC \cong \triangle A'B'C'$.*

Axiom 3. *(SAS Rule). If $AB = A'B', AC = A'C'$ and $\angle A = \angle A'$ then $\triangle ABC \cong \triangle A'B'C'$.*

Isosceles Triangle

Recall that the triangle $\triangle ABC$ is called isosceles if $AB = BC$.



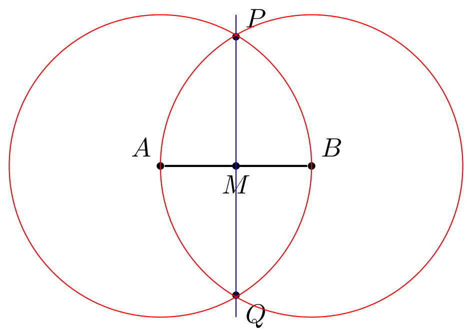
Theorem 1. *Properties of an isosceles triangle:*

1. *In an isosceles triangle, base angles are equal: $\angle A = \angle C$.*
2. *In an isosceles triangle, let M be the midpoint of the base AC. Then line BM is also the bisector of angle B and the altitude: BM is perpendicular to AC.*

Example: Finding the midpoint of the line segment

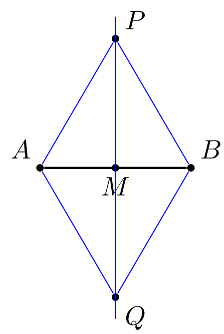
Problem: given two points A, B, construct the midpoint M of the segment AB.
Construction:

1. Draw a circle with center at A and radius AB
2. Draw a circle with center at B and radius AB
3. Mark the two intersection points of these circles by P, Q
4. Draw line through points P, Q



5. Mark the intersection point of line PQ with line AB by M. This is the midpoint.

Analysis: This is a two-step argument. In this figure, triangles $\triangle APQ$ and $\triangle BPQ$ are congruent (*why?*), so the corresponding angles are equal:



From this, we can see that $\triangle APM \cong \triangle BPM$, so $AM = BM$.