MATH 6. HW 24: INVARIANTS

1. Numbers 1 through 20 are written on the blackboard. Every minute two of the numbers are erased and replaced by their sum. Can you predict which number will be written on the board at the end?

2. Students have written on the blackboard 2011 "+" signs end 2011 "-" signs. Every minute a pair of signs is erased and replaced by a single "+" if they were equal or a single "-" if they were different. Can you predict which sign will be written on the board at the end? [Hint: look at the product]

3. Numbers 1 through 20 are written on the blackboard. Every minute a pair of numbers a, b are erased and replaced by $a + b - 1$. Can you predict which number will be written on the board at the end?

4. 6 trees grow in a row, with a distance of 10 meters between them. One bird sits on each tree. When one bird flies to a different tree, another bird flies in an opposite direction by the same distance. Can all birds get together on one tree?

5. There are 16 glasses on a table, arranged in a 4x4 grid, the glass in the bottom-left corner upside down. You are allowed to turn over any 2x2 square of glasses at a time. Can you get all the glasses standing correctly except the one in the top-right corner?

6. Is it possible for 17 people to be facebook friends with each other in such a way that each person is friends with exactly three other people in the group? [Hint: how many friendships would there be?]

7. In the country of RGB, there are 13 red, 15 green and 17 blue chameleons. Whenever two chameleons of different colors meet, both of them change their color to the 3rd one (e.g., if red and green meet, they both turn blue). Do you think it can happen that after some time, all chameleons become the same color?[Hint: give each color a numeric value, say 0, 1, 2]

8. A band of four thieves are wandering a city that has a perfect square grid road plan. The thieves always face along the direction of one of the grid lines; at any time, they may walk forward to the next intersection in the direction they are facing, or they may turn 90 degrees to face a different direction. Whenever they perform one of these actions, they plunder a silver coin.

The thieves are initially positioned at the corners of downtown, which is itself the shape of a square, and begin with no silver coins. Is it possible for them to meet up at the same intersection facing the same direction in such a way that they plunder a total of an odd number of silver coins?

9. There are 16 glasses on a table, arranged in a 4x4 grid, the glass in the bottom-left corner upside down. You are allowed to turn over any 1x4 row or column of glasses at a time. Can you get all the glasses standing correctly except the one in the top-right corner?