MATH 6: HOMEWORK 22: ODD AND EVEN

1. In a certain country the parliament has 400 members. After voting for a certain bill, the chairman announced that the bill was approved: there were zero abstentions, and 27 more votes in favor of the bill than against. The opposition claims that the voting results were falsified. Why?

2. A bag of 300 hundred gold coins has only coins with values of 1, 3, 5 and 15 piasters. The note on the bag says that the total is 1001 piaster. Can it be correct?

3. The numbers 1 through 10 are written on the blackboard in a row, with spaces left between them. Adam and Bill are playing the following game: on his turn, each player puts either + or - between two numbers. After all signs are written (so they get something like $1 + 2 - 3 + 4 + 5 - 6 \ldots$), the total is computed. If it is even, Adam wins; if it odd, Bill. What is the best strategy for Adam? should he take the first turn or leave the first turn to Bill?

4. A grasshopper is jumping along the number line: the first jump is 1 cm long, the second one, 2 cm, and so on. Can he return to his starting position after 9 jumps? 10 jumps? 2021 jumps?

5. The numbers 1 through 6 are written on the board. You can add 1 to two of the numbers. By repeating this many times, can you make all numbers equal?

6. Can you connect 2021 computers with cables so that each computer is connected to exactly 3 other ones?[**Hint:** how many cables you would need?]

7. A train consists of a locomotive and five cars marked I, II, III, IV and V. In how many ways can you rearrange the cars, in such a way that car I is always closer to the locomotive than car II?

8. Prove the following: *There were once n armed Robbers $R_1, \ldots, R_N$. Every time two Robbers meet one another, one of them robs another and takes all his arms. The Robber which has been already robbed cannot rob anybody (for he has no arms). Suppose that during some time period every two Robbers have met one another. Then by the end of this period, all the arms will be held by one and only one Robber.*

[I'm sure you understand why it's true. But try to write a proof – try to contradict their being more than one Robber at the end.]

9. * Suppose you want to find the midpoint between two points A and B, but you only have a compass. How can you do it? [The way I know how requires draw 8 circles].