MATH 6: HOMEWORK 19
ARITHMETIC SEQUENCES

1. Given some integer r and arithmetic sequence $a_1, a_2, a_3 \ldots$, let $b_1 = r \cdot a_1$, $b_2 = r \cdot a_2$, $b_3 = r \cdot a_3$ ... . Prove that $b_1, b_2, b_3 \ldots$ is an arithmetic sequence.

2. Prove that, given any arithmetic sequence, if I multiply each term by the same number and then add the same number to each term, the result is still an arithmetic sequence.

3. Write out the first four terms of each of the following geometric sequences, given the first term b_1 and common ration q.

(a) $b_1 = 1$ and $q = 3$

(b) $b_1 = 1$ and $q = \frac{1}{2}$

(c) $b_1 = -10$ and $q = \frac{1}{2}$

(d) $b_1 = 27$ and $q = -\frac{1}{3}$

4. What are the first two terms of the geometric progressions $a_1, a_2, 24, 36, 54, \ldots$?

5. Find the common ratio of the geometric progressions $1/2, -1/2, 1/2, \ldots$. What is a_{10}?

6. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50th term?

7. Calculate:

$$S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^{10}}.$$

8. Calculate:

$$S = 1 - 2 + 2^2 - 2^3 + 2^4 - 2^5 + \cdots - 2^{15}$$

9. Calculate:

$$1 + x + x^2 + x^3 + x^4 + \cdots + x^{100}$$

10. Calculate

$$S = 1 + 3 + 9 + 27 + 81 + 243,$$

first via the method of multiplying by the common ratio, then by plugging into the formula directly. Which method do you like better?

11. Calculate the infinite sum:

$$S = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \ldots.$$