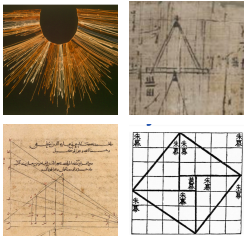


Numbers and Counting



Pythagoras(~600BC?) The Pythagoreans

Who was Pythagoras?

Pythagoras (~600BC?) - The Pythagoreans

- We know almost nothing about Pythagoras.
- Some scholars even doubt he existed. Most believed he as a Holy Man.
- The Pythagoreans form a very secretive sect.
- The mathematical and the mystical were merged.



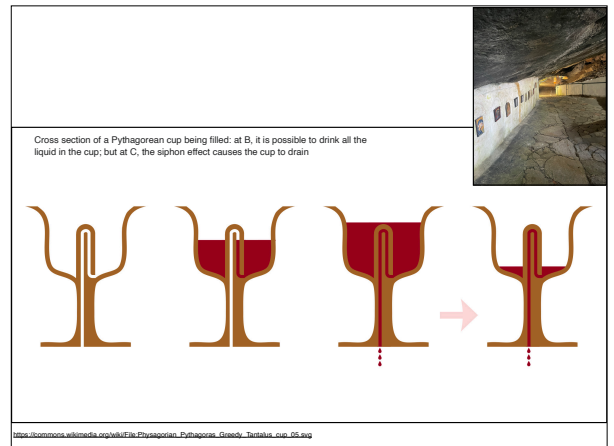
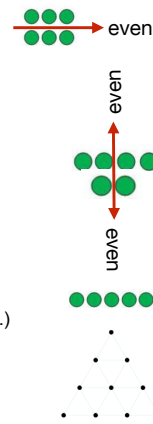
Doll by UnseekDollDesigns

All things are
number

No primary sources 🤔

The Pythagoreans

- Recall Dichotomy between odd and even.
- Pythagoreans probably represented numbers with pebbles.
 - **Example:** A number is **even** if it can be represented by a configuration of pebbles that than be divided into two equal parts. Otherwise is odd. (Well, 1 was not considered odd, nor even)
 - Proof: $4+2$ is even.
 - Proof: An even sum of odd numbers is even. (Similar to the proof of $4+2$ is even.)



**What is
mathematics?**

**What is
mathematics?**

**What do you mean
by mathematics?**
Fundamental type of question

**What is
counting?**

**What is
counting?**

**What do you mean
by counting?**

**What is counting?
Explain it to
someone who does
not know what it is**

**By counting one (usually)
means a process to determine
the number of objects in a set.**

**Counting involves establishing a one-to-
one correspondence between two sets.**

- One set (let's call it the *standard set*) is a list of symbols, objects, etc.
- The other set is variable.



Image credit: <https://creativelivingparentingblog.com/one-to-one-correspondence-intervention-for-kindergarten/>

Counting
If the standard set is composed of symbols, then each symbol

- has to be different from the others.
- has a prescribed order of use.

Body parts as numerals, by Geoffrey Saxe. <https://www.jstor.org/stable/pdf/1129244.pdf>

Counting

Tally

Early Latin *talea* = a cutting, rod, or stick

Late Latin *taliare* = to cut or split (same root as tailor)

Determine how many

Record how many

- Objects:
 - sticks, rocks shells - Abacus
- Body parts:
 - fingers, toes

assigning labels, language

Counting and the brain

Counting and the human brain

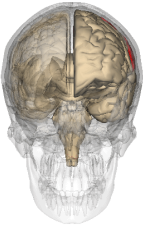
- Humans have innate number sense
- Small number words come from body parts
- Brain connects finger control with number processing

Hypothesis: Evolutionary link between fingers and counting

Number sense is a short-hand for our ability to quickly understand, approximate, and manipulate numerical quantities

For more listen to the podcast: <https://programmes/p00543nk> - with Ian Stewart and

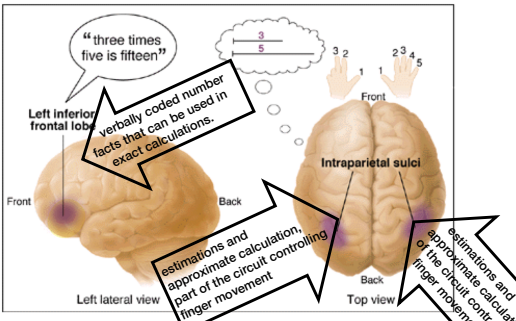
Parietal lobe (red) in left hemisphere,



from Anatomography, website maintained by Life Science Databases (LSDb).

Numbers and the brain

Brain Buttenworth: A Head for Figures. Science 294(553): 925-928, DOI:10.1126/science.294.5416.925



Left inferior frontal lobe

verbally coded number facts that can be used in exact calculations.

Front

Back

Left lateral view

estimations and approximate calculation, part of the circuit controlling finger movement

Intraparietal sulci

Front

Back

Top view

estimations and approximate calculation, part of the circuit controlling finger movement

Approximate number system - Object tracking system

Two separate mental systems to represent number without symbols in humans and many animals

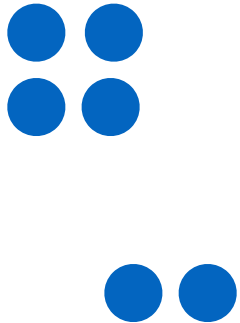
Approximate number system:

- approximate
- non-verbal
- Represents and compares numerosities
- ratio is key.



Example: Experiment where a monkey selects between two groups of dots displayed on a screen, choosing the group with the smaller quantity.

Two separate mental systems to represent number without symbols in humans and many animals



Object tracking system: allows grasping only small set sizes, from one to about four, in an unconscious but relatively precise way.

Note: Grasping (not counting!) up to four.

Two separate mental systems to represent number without symbols in humans and many animals



Just to illustrate that animals have also numbers sense (although feeding their chicks involves many systems...)

Object tracking system: allows grasping only small set sizes, from one to about four, in an unconscious but relatively precise way.

Note: Grasping (not counting!) up to four.

Two separate mental systems to represent number without symbols in humans and many animals

Approximate number system:

- approximate
- non-verbal
- Represents and compares numerosities
- ratio is key.

Object tracking system: allows grasping only small set sizes, from one to about four, in an unconscious but relatively precise way.

Note: Grasping (not counting!) up to four.

Clever Hans, the horse who could count?

- Around 1900 Von Osten claimed his horse, Clever Hans could do arithmetic.
- Von Osten would show Clever Hans written problems or ask questions aloud
- The horse would tap out the answer with its hoof.



Animals do have counting abilities but sometimes these are exaggerated!!



Examples of counting in different societies

**Can counting happen
without words for
numbers? If so,
how? If not, why?**

Example: Veddas of Sri Lanka counting

- Lived by eating fruit and turners of jungle plants.
- Had few words to deal with numbers. Some of these words (translated) were: a single, a couple, another one and many.
- To count, say, a pile of coconuts, a Veda collected a **heap of sticks**. Then,
 - For each coconut in the pile, she took a stick.
 - Each time she took a stick, she said "another one"
 - When finished, she pointed out the pile of sticks she took and said "That many".

Veddas count does not use number-words

https://sites.utexas.edu/dsh/files/2012/10/Dorenas_Tennesson-Thompson-Hew-Ancient-Accountants-Invented-Numbers.pdf

- Begins with the thumb on one hand
- Enumerates 27 places around the body,
- Ends on the little finger of the opposite hand.
- To indicate a particular number, one points to the appropriate body part and says the body part name.
- Example: to indicate the number 12, one points to the ear which is the 12th body part and says the word for ear, "nata".

This way of counting is called body counting

Forms of body counting were quite spread around the globe as late as the 1800s

The Okapiam 27-body part counting system.
Image credit: Making Change in Okapiam Tradestores by Geoffrey B. Saxe and Indigo Esmonde

"What is your age in Oksapmin body counting? Use the corresponding body part to describe it in English. For example, if you are five years old, you would write 'pinky finger,' and if you are twenty-seven, you would write 'other pinky finger'."

The diagram illustrates the Oksapmin 27-body part counting system. It features a central figure of a person with 27 numbered points of interest. The numbering starts at the right thumb (1) and proceeds in a continuous, winding path around the body, including the head, back, and legs, ending at the left thumb (27). The path is indicated by arrows. The diagram is set against a yellow background.

Oksapmin from Papua New Guinea body counting

The Oksapmin 27-body part counting system.

Image credit: Making Change in Oksapmin Tradestores by Geoffrey B. Saxe and Indigo Esmonde



History of mathematics hidden in language

How can we explore the origins of numbers?

Some approaches:

- Comparing past and present number systems
- Analyzing notations in ancient number systems
- Investigating the origins and evolution of number words

From Two to Three: A Giant Leap

Three means "many" in many languages:

- English: "three" and "through"
- French: "trois" and "très" (very)
- Latin: "trans" (beyond) suggests surpassing a limit

Evidence in ordinal numbers:

- "First" = before all others
- "Second" = "the other" (Latin: following)
- "Third, fourth, fifth..." = systematic counting

The struggle: Moving from "few" to "many"

Early word numbers: Observe patterns

Num ber	English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
1	one	ains	unus	heis	un	eka	bat
2	two	twai	duo	dyo	dau	dva	biga
3	three	threis	tres	treis	tri	tri	hirur
4	four	fidwor	quattuor	tettares	pedwar	catur	laur
5	five	fimf	quinque	pente	pump	panca	bortz
6	six	saihs	sex	hex	chwech	sad	sei
7	seven	sibun	septem	hepta	saith	sapta	zazpi
8	eight	ahtau	octo	okto	wyth	asta	zortzi
9	nine	niun	novem	ennea	naw	nava	bederatzi
10	ten	taihun	decem	deka	deg	dasa	hamar

A counting system is quite a complex construction, with different historical structures overlaying one another

Describe all patterns you can find in this table

Describe all patterns you can find in this table

Num ber	English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
11	eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
12	twelve	twalif	duodecim	dodeka	deudddeg	dvaadashan	hamabi
13	thirteen	prithund	tredecim	treiskaideka	tri ar ddeg	trayodasha	hamahiru
14	fourteen	fidwartehund	quattuordecim	tessareskaideka	pedwar ar ddeg	chaturdasha	hamalau
15	fifteen	fimftehund	quindecim	pentekaideka	pymtheg	panchadasha	hamabost
16	sixteen	saihstehund	sedecim	hexkaideka	un ar bymtheg	shodasha	hamasei
17	seventeen	sibuntehund	septendecim	heptakaideka	dau ar bymtheg	saptadasha	hamazazpi
18	eighteen	ahtautehund	duodeviginti	oktokaideka	deunaw	ashtadasha	hemezortzi
19	nineteen	niuntehund	undeviginti	enneakaideka	pedwar ar bymtheg	navadasha	hemeretzi
20	twenty	twaltiglus	viginti	eikosi	ugain	vimsath	hogoi

Hidden History in Number Words

Num ber	English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
11	eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
12	twelve	twalif	duodecim	dodeka	deudddeg	dvaadashan	hamabi
13	thirteen	prithund	tredecim	treiskaideka	tri ar ddeg	trayodasha	hamahiru
14	fourteen	fidwartehund	quattuordecim	tessareskaideka	pedwar ar ddeg	chaturdasha	hamalau
15	fifteen	fimftehund	quindecim	pentekaideka	pymtheg	panchadasha	hamabost
16	sixteen	saihstehund	sedecim	hexkaideka	un ar bymtheg	shodasha	hamasei
17	seventeen	sibuntehund	septendecim	heptakaideka	dau ar bymtheg	saptadasha	hamazazpi
18	eighteen	ahtautehund	duodeviginti	oktokaideka	deunaw	ashtadasha	hemezortzi
19	nineteen	niuntehund	undeviginti	enneakaideka	pedwar ar bymtheg	navadasha	hemeretzi
20	twenty	twaltiglus	viginti	eikosi	ugain	vimsath	hogoi

- "Eleven" = "one left" – words close in English and Gothic.
- "Twelve" = "two left" – words close in English and Gothic.

We can infer that in here were northern European tribes whose counting words went up only to 'ten' then counted leftovers

Hidden History in Number Words

Base-20 Traces

- French: "quatre-vingts-sept" = $4 \times 20 + 7 = 87$
- English (Bible Psalm 90:10): "three score and ten" = $3 \times 20 + 10 = 70$

Mixed systems show historical layers

Can you remember a historical occurrence of the word "score"?
(Where score mean 20)





10101101

Number Systems

Number systems

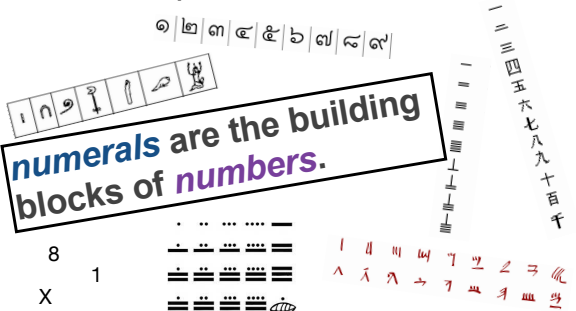
A **number system** consist on a set of symbols, called **numerals**, and a set of **rules** for writing this numerals to represent **numbers**.

Give examples in each cell of the table

number system	numerals	numbers	rules
Hindu-Arabic ("ours")	0,1,	123410	
Roman	I, V, X...	XXX IV	
Binary	0,1	100010	

Number systems

A **number system** consist on a set of symbols, called **numerals**, and a set of **rules** for writing this numerals to represent **numbers**.



numerals are the building blocks of numbers.

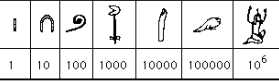
Additive number systems

Number systems

A **number system** consist on a set of symbols, called **numerals**, and a set of **rules** for writing this numerals to represent **numbers**.

We are going to consider four **characteristics** of number systems

- Additive: The value of a number is the sum of the values of the numerals.
- Ciphered or alphabetic
- Multiplicative
- Positional

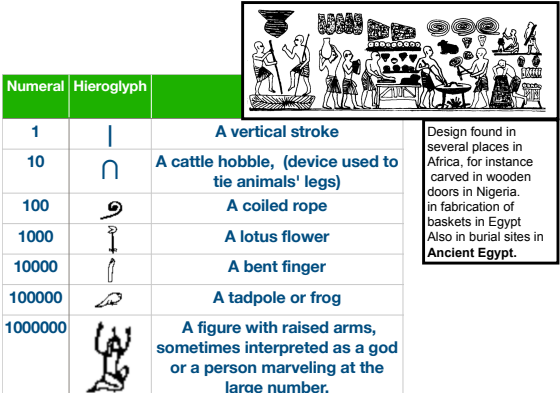


1	10	100	1000	10000	100000	10 ⁶
---	----	-----	------	-------	--------	-----------------

Egyptian numeral hieroglyphs

Egyptian Additive Number System

Additive: The value of a number is the sum of the values of the numerals.



Design found in several places in Africa, for instance carved in wooden doors in Nigeria. in fabrication of baskets in Egypt Also in burial sites in Ancient Egypt.

An additive number system: Egyptian Hieroglyphs

Images credits: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

- based on a **scale** of 10
- used as far back as 3400 B.C.E.
- mostly for inscription in stones

numerals:

1	10	100	1000	10000	100000	10 ⁶

Egyptian numeral hieroglyphs



1. Write the number 752 in Egyptian hieroglyphics.
- 2 Express number on the left in Hindu-Arabic numerals.

An additive number system: Egyptian Hieroglyphs

Images credits: https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

- based on a **scale** of 10
- used as far back as 3400 B.C.E.
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numerals:

1	10	100	1000	10000	100000	10 ⁶

Egyptian numeral hieroglyphs

Example

276

- Educated guess: what are the rules of the Egyptian hieroglyphic system?**
- Hint: One rule is related to the number of times a numeral can be repeated.

An additive number system invented by your instructor

value	1	5	25	125
numerals	a	b	c	d

1. Express abbcdd in Hindu-Arabic numerals.
2. Express 106 in this additive system
3. What are the disadvantages of an additive number system?

Rules:

- Numerals are written from left to right, from the numeral with smallest value to the numeral with largest value. (abbcdd)
- The number of numerals used must be the smallest possible (for instance, we should write "b" instead of "aaaa")

Ciphered or alphabetic number systems

Number systems

A **number system** consist on a set of symbols, called **numerals**, and a set of **rules** for writing this numerals to represent **numbers**.

A number system can be.

- Additive:
- Ciphered or alphabetic: Numerals design 1, 2,...9, and the powers of 10 (or, more general some base) but also to the multiples of this powers. Example: Greek Alphabetic
- Multiplicative
- Positional:

Letter	Value	Letter	Value	Letter	Value
α	1	ι	10	ρ	100
alpha		iota		rho	
β	2	κ	20	σ	200
beta		kappa		sigma	
γ	3	λ	30	τ	300
gamma		lambda		tau	
δ	4	μ	40	υ	400
delta		mu		upsilon	
ϵ	5	ν	50	ϕ	500
epsilon		nu		phi	
ζ	6	ξ	60	χ	600
digamma		xi		chi	
ζ	7	$\omicron$	70	ψ	700
zeta		omicron		psi	
η	8	π	80	ω	800
eta		pi		omega	
θ	9	$\koppa$	90	$\sampi$	900
theta		koppa		sampi	

Table from <https://online.math.uh.edu/Math2303-unpaiddict/1/s12/index.html>

A ciphered number system: Greek Alphabetic

Numerals

Rule: Numeral in ascending value, from right to left. Repetitions?

1. Write the number 752 in Greek numerals
2. Translate $\sigma\pi\gamma$ to Hindu-Arabic.

Letter	Value	Letter	Value	Letter	Value
α	1	ι	10	ρ	100
alpha		iota		rho	
β	2	κ	20	σ	200
beta		kappa		sigma	
γ	3	λ	30	τ	300
gamma		lambda		tau	
δ	4	μ	40	υ	400
delta		mu		upsilon	
ϵ	5	ν	50	ϕ	500
epsilon		nu		phi	
ζ	6	ξ	60	χ	600
digamma		xi		chi	
ζ	7	$\omicron$	70	ψ	700
zeta		omicron		psi	
η	8	π	80	ω	800
eta		pi		omega	
θ	9	$\koppa$	90	$\sampi$	900
theta		koppa		sampi	

Table from <https://online.math.uh.edu/Math2303-unpaiddict/1/s12/index.html>

Hieratic script is the cursive form of hieroglyphic. It was used for administrative and literary purposes.

What type of number system is the hieratic one? (additive, multiplicative, ciphered, positional) Why?

1	𐩀	10	𐩁	100	𐩂	1000	𐩃
2	𐩄	20	𐩅	200	𐩆	2000	𐩇
3	𐩈	30	𐩉	300	𐩊	3000	𐩋
4	𐩌	40	𐩍	400	𐩎	4000	𐩏
5	𐩐	50	𐩑	500	𐩒	5000	𐩓
6	𐩔	60	𐩕	600	𐩖	6000	𐩗
7	𐩘	70	𐩙	700	𐩚	7000	𐩛
8	𐩜	80	𐩝	800	𐩞	8000	𐩟
9	𐩠	90	𐩡	900	𐩢	9000	𐩣

Hieratic numerals

https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_numerals/

Greek alphabetic numerals

Letter	Value	Letter	Value	Letter	Value
α alpha	1	ι iota	10	ρ rho	100
β beta	2	κ kappa	20	σ sigma	200
γ gamma	3	λ lambda	30	τ tau	300
δ delta	4	μ mu	40	υ upsilon	400
ε epsilon	5	ν nu	50	φ phi	500
ς digamma	6	ξ xi	60	χ chi	600
ζ zeta	7	ο omicron	70	ψ psi	700
η eta	8	π pi	80	ω omega	800
θ theta	9	Ϟ koppa	90	Ϡ sampi	900

Does the Greek alphabetic system count as additive? Explain why or why not.

A multiplicative system
Traditional Chinese numerals

Write the numbers below (in traditional Chinese numerals) in Hindu-Arabic numerals.

(a) 八十三 (b) 四百七十 (c) 二萬九千五 (d) 五千六百三十四

1	一	10	十
2	二		
3	三	100	百
4	四		
5	五	1000	千
6	六		
7	七	10,000	萬
8	八		
9	九		

Burton, David M. "The history of mathematics: An introduction." (1985)

Burton, David M. "The history of mathematics: An introduction." (1985)