

Describe all patterns you can find in this table

Number	English	Gothic	Latin	Ancient Greek	Welsh	Sanskrit	Basque
11	eleven	ainlif	undecim	hendeka	un ar ddeg	ekaadasha	hamaika
12	twelve	twalif	duodecim	dodeka	deudddeg	dvaadashan	hamabi
13	thirteen	þritenhund	tredecim	treiskaideka	tri ar ddeg	trayodasha	hamahiru
14	fourteen	fidwartehund	quattuordecim	tessareshkaideka	pedwar ar ddeg	chaturdasha	hamalau
15	fifteen	fimftehund	quindecim	pentekaideka	pymtheg	panchadasha	hamabost
16	sixteen	saihtehund	sedecim	hexkaideka	un ar bymtheg	shodasha	hamasei
17	seventeen	sibuntehund	septendecim	heptakaideka	dau ar bymtheg	saptadasha	hamazazpi
18	eighteen	ahtatehund	duodeviginti	oktakaideka	deunaw	ashtadasha	hemezortzi
19	nineteen	niuntehund	undeviginti	enneakaideka	pedwar ar bymtheg	navadasha	hemeretzi

Number systems

A **number system** consist on a set of symbols, called **numerals**, and a set of **rules** for writing this numerals to represent **numbers**.

Give examples in each cell of the table

number system	numerals	numbers	rules
Hindu-Arabic			
Roman			
Binary			

A number system can be:

Additive: The value of a number is the sum of the values of the numerals.

Ciphred or alphabetic: Numerals design 1, 2,..9, and the powers of 10 (or, more generally, some base) but also to the multiples of this powers.

Example: Greek Alphabetic

Multiplicative: There are two sets of numerals, the elements of one set represent digits and the elements of the other set represent position. If necessary, a digit and a position symbols are used together, and the values of numerals are multiplied. Finally, all the products are added.

Positional: The value of each numeral depends on its position. The system consists of a base (a natural number greater than one) and a set of numerals representing the numbers from zero to one less than the base. The numbers from zero to the base minus one are the digits in the system.

An additive number system: Egyptian Hieroglyphs

- based on a **scale** of 10
- used as far back as 3400 B.C.E.
- mostly for inscription in stones



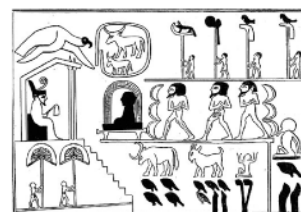
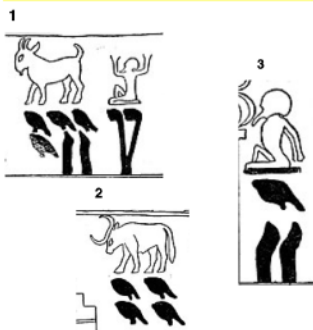
- Write the number 752 in Egyptian hieroglyphics.
- Express number on the left in Hindu-Arabic numerals.

numerals:

1	10	100	1000	10000	100000	10 ⁶

Egyptian numeral hieroglyphs

Decipher 1, 2 and 3 (on the left).



Narmer Macehead (drawing). The design shows captives being presented to Pharaoh Narmer, enthroned in a boat. Ashmolean Museum, Oxford.

Educated guess: what are the rules of the Egyptian hieroglyphic system?
Hint: One rule is related to the number of times a numeral can be repeated.

An additive number system invented by your instructor

value	1	5	25
numerals	a	b	c

- Express abbcdd in Hindu-Arabic numerals.
- Express 106 in this additive system
- What are the disadvantages of an additive number system?

Rules:

- Numerals are written from left to right, from the numeral with smallest value to the numeral with largest value. (abbcdd)
- The number of numerals used must be the smallest possible (in instance, we should write "b" instead of "aaaa")

A ciphered number system: Greek Alphabetic Numerals

Rule: Numeral in ascending value, from right to left. Repetitions?

- Write the number 752 in Greek numerals
- Translate στγ to Hindu-Arabic.

Letter	Value	Letter	Value	Letter	Value
α alpha	1	ι iota	10	ρ rho	100
β beta	2	κ kappa	20	σ sigma	200
γ gamma	3	λ lambda	30	τ tau	300
δ delta	4	μ mu	40	υ upsilon	400
ε epsilon	5	ν nu	50	φ phi	500
ς digamma	6	ξ xi	60	χ chi	600
ζ zeta	7	ο omicron	70	ψ psi	700
η eta	8	π pi	80	ω omega	800
θ theta	9	ϙ koppa	90	ςampi	900

Hieratic script is the cursive form of hieroglyphic. It was used for administrative and literary purposes.

What type of number system is the hieratic one? (additive, multiplicative, ciphered, positional) Why?

1	10	100	1000
2	20	200	2000
3	30	300	3000
4	40	400	4000
5	50	500	5000
6	60	600	6000
7	70	700	7000
8	80	800	8000
9	90	900	9000

Hieratic numerals

Does the Greek alphabetic system count as additive? Explain why or why not.

A multiplicative system Traditional Chinese numerals

Write the numbers below (in traditional Chinese numerals) in Hindu-Arabic numerals.

(a) 八十三 (b) 四百七十 (c) 二萬九千五百三十四 (d) 五十六百三十四

1	十
2	二十
3	三十
4	四十
5	五十
6	六十
7	七十
8	八十
9	九十

Invented multiplicative system

					1	10	100	1000	
numerals representing position					a	b	c	d	
Digits (numerals)	1	2	3	4	5	6	7	8	9

- Translate d2c7b3a8 from the multiplicative system to the Hindu-Arabic number system.
- Write 1065 down the numbers in the multiplicative system

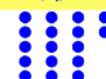
Important statement for Positional Number Systems

Integer division:

Given two integers a and b , with $b > 0$, there exist unique integers q and r such that $a = b \cdot q + r$ and $0 \leq r < b$

In this figure, $a=17$. What are the values of b , q , and r ?

- a is called the *dividend*,
- b is called the *divisor*,
- q is called the *quotient*,
- r is called the *remainder*.



This statement answers the question: What is the maximum number of times b "enters" into a , and what is remaining after this maximum number of b is subtracted from a ?

Note: The result works for a , b integers, $b \neq 0$, but we will only work with positive numbers.

From base 10 to base $b \neq 10$

We are given number N (in base 10). Suppose N has the form $N = (u, v)_b$, then we have $N = u \cdot b + v$, with $0 \leq u, v < b$. In this case, to write N in base b need to find u and v .

Find u and v when $N = 100$ and $b = 11$. Give your answer as an ordered pair (u, v)

Recall: Given two integers a and b , with $b > 0$, there exist unique integers q and r such that $a = b \cdot q + r$ and $0 \leq r < b$

All the Mayan numerals

0	1	2	3	4
	•	••	•••	••••
5	6	7	8	9
	•	••	•••	••••
10	11	12	13	14
	•	••	•••	••••
15	16	17	18	19
	•	••	•••	••••

- Express the number 752 in base 20.
- Express 20 in base 20.
- Express the numbers 445 and 752 in the Mayan number system. Mayan numbers are written vertically, with units at the bottom, multiples of twenty above, and so on.

Complete the table

Mayan	Hindu-Arabic ("ours") and Rod numerals	Roman	Egyptian hieroglyphics	Traditional Chinese	Greek alphabetic	Moir's Multiplicative system
	25					
	DCCXIX					
	100					